JUNIOR
HIGH SCHOOL
MATHEMATICS

BOOK
III

THEODORE LINDQUIST

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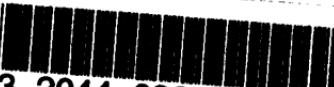
JUNIOR HIGH SCHOOL
MATHEMATICS

BOOK III

BY

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TO THE TEACHER

In presenting the two earlier books of this series, the author in each of them offered the teacher some reasons for developing the subject of mathematics for the Junior High School as he had developed it. It seems to be in place here to repeat some of the considerations and conclusions there expressed.

Because inductive geometry together with its accompanying constructions has been found to be the most concrete of the mathematics work that should be given to pupils of the Junior High School, it was stressed as the most important feature of Book I, which is intended for use in the seventh year. In order to lead the pupils to greater generalization and to further the use of mathematics as an instrument, literal numbers and formulas, and the application of tables were made the main issues of Book II, which is intended for pupils in the eighth year. Business principles and problems appear throughout the books for these two years. The principles are quite elementary and the problems deal almost wholly with thrift and with child enterprises.

The general business problem is concrete to the average grown person but becomes less and less concrete the younger a person is. The pupils of the ninth year look at life more nearly from the standpoint of the adult, and therefore may be depended upon to give far greater weight to these business matters than they would have given during either of the previous two years. Hence, the general business applications of mathematics have been delayed until the ninth

iii

year, and are therefore the substance of this, the third book of the series.

Placing the business applications in the third year of the Junior High School course is logical both from the pupil's needs and from the nature of the service to which mathematics can be put. Opportunity for the application of the principles studied the first two years now arises naturally and the pupils find real use for their mathematics.

The work in Book I and in Book II provides ample suggestions for project problems commensurate with the abilities of the pupils of those two years. The pupils of the ninth year, however, are able to handle the larger and more complex project problems that arise on every hand in the business applications of mathematics. Such problems are met, for instance, in the study of savings-banks, of transportation, and of the items in the family budget which properly may be discussed.

The above-outlined plan of placing the great bulk of the business applications of mathematics in the last year of the Junior High School course has been submitted by the author to a great number of men in business and industry as well as to educators, and they have given the plan their universal approval.

The pupils of the Junior High School may be divided into two large groups: first, those who leave school at the end of this period and, second, those who enter Senior High School. The course in mathematics should be so arranged as to prepare properly each of these groups of students, the first to go out into life, the second to continue their studies advantageously. This the author has continually kept in mind in developing the series. In order best to help the first group, the Junior High School mathematics needs to *be a well-rounded* course complete in itself. A part of the

plan of making the series a complete unit, reaching its goal at the proper time, is to give the business applications of the mathematics in the year just before the pupils leave school to use their mathematics in actual life.

Much attention has been given of late by educators and by the public at large to the matter of economy of time in our schools, and the pupil who continues his education by entering the Senior High School must also be considered in formulating the Junior High School mathematics course. Hence, the work of the ninth year as well as that of the previous two years has been so developed that the pupil entering Senior High School will neither miss any necessary work nor will he be compelled to lose time by repetitions. In Book III the application of the work taken in the previous two years together with the cumulative reviews takes care of these matters.

After completing the three books of Junior High School Mathematics, the pupil entering Senior High School will be able to take up third-semester algebra, to complete plane and solid geometry in one year, to complete trigonometry in one-half year, and to complete college algebra in one-half year.

In the third book, as in the first two books, the author has stuck to the principle of selecting material which naturally appeals to all of the pupils of the Junior High School, and which can be made of value to them all. Each topic is taken up almost wholly from the standpoint of the one outside and not inside of the counter. For instance, we all make deposits and withdrawals at the bank, but few of us are ever employed in a bank. Again, every one is interested in the development of transportation facilities, in freight and passenger rates, and in a variety of associated problems, but only a few of us are ever employed by a transportation

company. Hence, the processes and principles that will come within the range of the pupils of the Junior High School in general are the ones here included. All specializations should be excluded for present purposes.

In order to make this book useful in the greatest possible degree, many of the topics have been treated in a somewhat different manner from that found in most texts. For instance, in place of giving several differing processes of computing interest, some space is devoted to a discussion of investments. A consideration of the historical and economic phases of various topics arouses interest and gives a breadth of view which cannot be obtained by a purely mathematical treatment. In the discussion of the subject of wages, a study is made of the value of an education. This particular question, being placed at about the time when each pupil will be debating the advisability of entering Senior High School, becomes a real personal problem.

In compliance with the advice of men in various lines of business and industry, numerical computations have been given a prominent part. This includes the most useful processes of short cuts, approximations, and checks. The number contests employed during the first two years may be continued at the discretion of the teacher.

The author takes pleasure in acknowledging the inspiration and suggestions gathered from his students, his fellow teachers, and men of business and industry. He feels especially indebted to Miss Lena B. Hansen, of the Enid, Oklahoma, High School, and to his two colleagues, Mr. W. H. Keller and Miss Inez Morris.

<div align="right">THEODORE LINDQUIST.</div>

EMPORIA, KANSAS.

TABLE OF CONTENTS

JUNIOR HIGH. SCHOOL MATHEMATICS

BOOK III

I

REVIEW INTEGERS AND DECIMALS

1. Need of Business Mathematics.—A building contractor made a mistake in his estimate of the cost of remodelling the high-school building of his city. One of his computations came out $ 55 instead of the correct amount $ 550. How much too small was his estimate because of this error? His bid to the Board of Education was about $ 600 lower than the next lowest bid and he was awarded the contract. If he had not made the error, would his bid still have been the lowest? How. much did he lose through his error?

A lady bought a chicken weighing 3½ lb. at 25 ¢ per pound. How much should she have paid for it? She tendered a dollar bill in payment and received 2 ¢ in change without noticing any error. How much would she lose during the year, if she averaged two such errors per week?

Accuracy is the first and the last requirement in all business—accuracy as to how to proceed and also accuracy in the process. System and neatness are highly important in securing accuracy as well as in saving time.

1

2. Reading Numbers.—Numbers should be read quickly but accurately and so that they can be copied readily. For writing in a column read 5436.28,   5436.28 "fifty-four, thirty-six, point, twenty-eight";   563.75 563.75, "five hundred sixty-three, point, seventy-   2607.00 five"; 2607.00, "twenty-six, oh, seven, point,   6000.45 oh, oh," or "double oh" for 00; 6000.45, "sixty, double oh, point, forty-five," or "six thousand, point, forty-five." Use **and** for the decimal point only.

3. Writing Numbers.—Clear, well-made figures are necessary for accurate computations. In writing numbers in words, as on checks, use a capital only in beginning the first word. Use the hyphen in all compound words, as twenty-five.

EXERCISES

1. Read the numbers in the first column on page 7.

2. Read as telephone numbers: 2364, 4506, 4067, 300.

3. Write in figures: one thousand six hundred thirty; five hundred forty-seven; three thousand nine hundred fifteen; sixteen thousand five hundred nine; twenty thousand ninety-seven; ninety-two thousand three hundred fifty-eight; three thousand seventy-one; three hundred forty-nine and thirty-five hundredths.

4. Write a list of numbers at the teacher's dictation.

5. Write the following in words: 43, 76, 248, 1345, 230, 5.28, 30.43, 10.09.

4. United States Money.—The United States Government issues the following coins:

1. Gold: double eagle, eagle, half-eagle, quarter-eagle.
2. Silver: dollar, half-dollar, quarter-dollar, dime.
3. Nickel: five-cent piece.
4. Copper: one-cent piece.

The eagle is seldom mentioned in business.

It also issues the following paper money, or currency:

1. Gold certificates: secured by gold in the United States Treasury.

2. Silver certificates: secured by silver in the United States Treasury.

3. United States notes (greenbacks): promissory notes by the United States to pay a stated number of gold or silver dollars.

4. National bank notes: promissory notes, promising to pay on demand, issued by national banks through the government.

5. Federal-reserve bank notes: notes issued by the twelve federal-reserve banks similiar to those issued by the national banks.

The denominations of paper money now in circulation are: $ 1, $ 2, $ 5, $ 10, $ 20, $ 50, $ 100, $ 500, $ 1000, and $ 10,000

5. Reading and Writing United States Money.—United States money is written as dollars and decimal parts of a dollar. Thus, $ 45.35, forty-five and $\frac{35}{100}$ dollars. 56 cents is written $ 0.56, $ .56, 56 ¢, or 56 cts. Why is $ 0.56 better than $ .56 ?

EXERCISES

1. State the largest piece of money that you would use in paying each of the following: $ 45.35; $ 125.47; $ 17.34.

2. Write in words the following: $ 45.36; $ 172.95; $ 105.23; $ 1005.25; $ 1250.50.

3. Give the table for United States money. Which unit is not coined ?

4. Why is the purchasing power of a $ 5 bill of any of the above issues the same as of a $ 5 gold piece ?

5. When did the paper money of the United States not have the same purchasing power as coins of the same denomination ? Can you give a reason for this ?

6. What is the origin of "Not worth a Continental " ?

7. Do you know of a country where the purchasing power of paper money is less than of the corresponding coins ? Can you give any reason for this ?

6. Addition.—A thorough knowledge of the 45 addition combinations is necessary to accuracy and rapidity in adding. Practise on any that may give you trouble.

EXERCISES

1. For the following, give the sum of each digit and the one to the right; to the left. Give the sum of each digit and the one above; the one below.

7	8	9	5	7	4	6	3	6	8
5	6	4	6	6	8	5	9	4	6
5	9	4	7	5	8	6	4	7	8
9	3	7	5	8	4	6	6	8	5
4	7	5	6	8	7	7	5	2	4
7	5	3	9	5	8	6	4	9	7
6	4	9	5	6	7	8	6	5	4
8	5	4	7	9	8	7	5	7	6
3	7	6	5	8	6	4	9	5	7
9	5	7	8	5	7	9	6	7	8

2. Add each of the above columns; add upward and downward to check your work.

7. Adding Machines. —In establishments in which much adding is done, as in banks, adding machines are often used. How could you make a mistake with the adding machine?

8. Rapid Column Addition.—One way to add rapidly is to look for combinations that add up to 10; also for digits that repeat.

2-4 3

```
5 2 8-7-
4 3-9-3
6-1 2 3
8-5 7 5-
5 0 6-2
5-6-3 6-
8-2 4-4
  8 3 7 5
-----------
5 1 6 9 5
```

In the first column we have $7 + 3$, $3 + 5 + 2$, and $6 + 4$. Together with the remaining 5 these make 35. In the second column are $3 + 7$ (twice), $8 + 2$, and $6 + 4$, which with 9 make 49. In the third column $4 + 2 + 3 + 1$ and $5 + 2 + 3$ together with the remaining 6 make 26. The fourth column has three 8's which make 24 and three 5's which make 15. These with $2 + 4 + 6$ make 51.

Another useful plan is to begin at the top or bottom and add until a sum equal to 10 or between 10 and 20 is reached. Ten is then dropped and a dash placed by the figure in the column. Any remaining number is now added to the next figure until 10 or more is again reached.

In the above $5 + 4 + 6 = 15$. A dash is placed by the 6. Then 5, from the 15, added to 2, and 5 make 12. A dash is placed after 5 and the 2 added to $3 + 3 + 7$, giving 15. Another dash is placed after the 7, and a 5 placed below the line. A glance at the column shows how many are to be carried to the next column. This is 3, which is written above 10's column. The remaining columns are added in like manner. Carry out the additions in these columns.

It is well to place the numbers to be carried above the columns. Especially is this useful for those who may be interrupted while adding.

Check all additions by adding upward and downward. Addition may also be checked by casting out the 9's, as will be explained later.

Make up several columns of numbers like the one above, and add in the manner there suggested.

9. Horizontal Addition.—It is often necessary to add horizontally across the page, as in footing up statistics. When adding the units be careful not to add in any of the tens; when adding the tens not to add in any of the units or hundreds; and so on. This is one of the main sources of error in horizontal addition.

<div align="center">EXERCISES</div>

1. Add horizontally each row in Ex. 1 on page 4. Also add each vertical column. Finally, add the new column obtained by the horizontal additions and the new row obtained by the vertical additions. What should you find if the additions are correct?

In the following three exercises carry out the same additions as for Ex. 1 on page 4.

2.	5647	8647	5946	6785	8567	9547	????
	5867	6587	5478	6847	8478	9785	????
	6847	7869	6787	9564	8576	5846	????
	7854	5674	5764	5789	5674	5867	????
	9647	6579	4839	5768	4595	4659	????
	9564	4764	9576	9574	4867	7456	????
	7885	6547	8758	6984	7785	9678	????
	8496	7864	5476	5787	6569	6758	????
	5978	8956	8675	6875	8975	5767	????
	????	????	????	????	????	????	????

3.	6784	9557	9784	4607	9368	7865	????
	5067	4867	5406	5917	4573	9456	????
	8587	7438	9673	6174	7394	7673	????
	6039	5473	7568	7514	8653	6854	????
	4567	7485	8567	8647	5769	8574	????
	7936	9736	6758	5906	7485	6968	????
	8487	6957	8596	7439	5837	9475	????
	6759	5474	5978	6857	9485	4897	????
	????	????	????	????	????	????	????

4. 4560.45	3705.47	3494.06	??????
3053.74	4795.08	7830.45	??????
894.06	5790.57	5067.84	??????
4709.75	7437.60	6709.07	??????
6370.47	3075.06	· 7560.58	??????
797.68	5909.87	7560.58	??????
6068.57	6780.95	8705.87	??????
9507.80	8365.78	4657.60	??????
6408.75	7849.06	5076.08	??????
????.??	????.??	????.??	??????

5. Make up other columns and add in the same manner.

10. Excesses of 9's.—If a, b, and c are the digits of a number, then the number is $100a + 10b + c$. Why?

Also, $100a + 10b + c = 99a + 9b + (a + b + c)$. How? As $99a + 9b$ divided by 9 gives no remainder, any remainder comes from dividing $a + b + c$ by 9. But this remainder will be the same as dividing the number itself, $100a + 10b + c$ by 9. Why? Hence, dividing a number and the sum of its digits by 9 gives the same remainder. This remainder is called the *excess of 9's* of the number.

Digits whose sum is 9 give no remainder when divided by 9 and can be disregarded. In 845,633, $4 + 5 = 9$, and $6 + 3 = 9$ may be disregarded. If an excess is above 9, find its excess; that is, add the digits. Thus, excess of 83 is $8 + 3 = 11$, but excess of 11 is $1 + 1 = 2$. Hence, excess of 83 is 2.

EXERCISES

1. If the following numbers are represented by $100a + 10b + c$, find the value of a, b, and c for each: 234, 305, 713, 340, 200, 908.

2. Show that any number with 4 digits can be represented by $1000a + 100b + 10c + d$.

3. Find the excess of each of the following numbers: 2845, 9623, 4815, 7329, 6445, 8236, 4951, 5,623,741.

11. Checks.—Checking computations is highly important. Good checks must be simple to operate and must not be mere repetitions of the previous work. Excellent checks are based upon excesses of 9's.

12. Checking Addition.—One plan is to add upward as well as downward. Another is to find the excesses of each number and then find their sum. The excess of this is then found and if it equals the excess of the sum of the numbers, the work is likely correct.

$$(100a + 10b + c) + (100x + 10y + z)$$
$$= 100(a + x) + 10(b + y) + (c + z) \qquad \text{How?}$$
$$= 99(a + x) + 9(b + y) + (a + b + c) + (x + y + z).$$
$$\text{How?}$$

The excess of the sum of the numbers on the left equals the excess of the numbers on the right. But the latter is the excess of $(a + b + c) + (x + y + z)$. Why? That is, the excess of the sum of two numbers equals the sum of their excesses.

The decimal point plays no part in the check and its position is not checked.

111 2

456.32	excess	2
732.19	excess	4
874.12	excess	4
326.58	excess	6
2389.21		16

The excess of both 16 and 2389.21 is 7. What is the conclusion? If some digits of the sum were too large and others as much too small, the excess would still be 7 and the error not be shown. Such errors seldom arise.

EXERCISES

1. Write a column of at least eight numbers with four digits in each number. Add and check.

2. Repeat for at least five more additions.

13. Subtraction.—The simplest form of subtraction is to find a number which added to the subtrahend gives the minuend. This is used in making change.

$4 + 3 = 7$; write the **3**.	637　excess **7**
$6 + 7 = 13$; write the **7**, and carry 1.	264　excess **3**
1 (carried) $+ 2 + 3 = 6$; write **3**.	$\overline{373}$　　$\overline{4}$

14. Checking Subtraction.—Subtract the excess of the subtrahend from the excess of the minuend. If this equals the excess of the difference, the subtraction is probably correct. In the subtraction above what is the excess of 373 ? What is the conclusion as to the work ? What error would this check not have shown ? Explain:

$(100a + 10b + c) - (100x + 10y + z)$

　$= 100(a - x) + 10(b - y) + (c - z)$

　$= 99(a - x) + 9(b - y) + (a + b + c) - (x + y + z).$

The excess of the difference of the numbers on the left equals the excess of those on the right. The latter is the excess of $(a + b + c) - (x + y + z)$. Why ? Hence, the excess of the difference of two numbers equals the difference of their excesses.

If the excess of the subtrahend is greater than the excess of the minuend, add 9 to the latter.

EXERCISES

How would you make change for the following:

1. $ 1.25 paid with $ 2.　**3.** $ 2.39 paid with $ 5.

2. $ 2.65 paid with $ 5.　**4.** $ 7.36 paid with $ 10.

Make the following subtractions and check:

5.	57.48	**7.**	41.56	**9.**	23.82	**11.**	61.07
	29.39		25.28		19.69		49.04
6.	61.34	**8.**	62.35	**10.**	71.34	**12.**	92.30
	58.27		45.53		56.52		35.50

13. At the beginning of business one day a merchant had a balance of $ 560.74 at his bank. During the day he gave out checks for $ 356.47 and made a deposit of $ 279.06. What was his balance the next morning?

14. A merchant began business one day with $ 57.36 on hand. His sales amounted to $ 108.74 and he paid out $ 47.18 during the day. How much should he have on hand at the end of the day?

15. A merchant began business one day with $ 35.47 on hand and ended with $ 105.20 on hand. His cash register showed that he had received $ 89.65 for sales and had paid out $ 19.67. Had he made change correctly? If not, what was his error?

16. One day a grain merchant shipped 860 bu. oats, 460 bu. corn, and 930 bu. wheat. The same day he bought 780 bu. oats, 490 bu. corn, and 970 bu. wheat. Did he have more grain at the beginning or at the end of the day?

17. It is 451 mi. from Chicago to Kansas City, 338 mi. from Kansas City to Dodge City, Kans., and 385 mi. from Dodge City to Denver. How far is Chicago from Denver?

18. How far is Kansas City from Denver?

19. How far is Chicago from Dodge City?

20. La Junta, Colo., is 991 mi. from Chicago. How far is Kansas City from La Junta?

21. Find the total sales of each saleslady each day for the week and of the department for the week:

	Miss Muir	Miss Holly	Miss Jones
Monday	$ 28.75	32.19	31.78
Tuesday	29.68	27.56	26.09
Wednesday	32.75	28.07	27.68
Thursday	31.98	29.67	30.91
Friday	28.15	30.58	29.71
Saturday	39.67	34.54	32.97

15. Multiplication.—Use the common form for multiplication of numbers with two or more digits. Speed and accuracy demand a thorough knowledge of the multiplication combinations. If you have trouble with any, practise on these until you can give them at once.

16. Powers.—The result found from multiplying a number by itself is spoken of as a **power** of that number. They are written thus—$(345)^2$, $(617)^3$, etc.—and are read the **second power** of 345, the **third power** of 617, etc.

17. Checks of Multiplication.—To check multiplication find the excess of the multiplicand and of the multiplier. Find the product of these two and then its excess. If this last equals the excess of the product of the two numbers, the multiplication is likely correct.

$$
\begin{array}{ll}
246 \ — & 3 \ \text{excess} \\
\underline{635} \ — & \underline{5} \ \text{excess} \\
1230 & 15 \ \text{excess } 6 \\
738 & \\
\underline{1476} & \\
156210 & \text{excess } 6, \text{ the same as above.}
\end{array}
$$

<div align="center">

EXERCISES

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1. What is the name of each of the two numbers multiplied together and of the result?

2. State the law for the placing of the decimal point in the multiplication of decimals.

3. Turn to page 4 and give the products of the numbers where the sums are called for. Practise on these until you can give them without the least hesitation.

4. Give in the simplest manner the excesses of each of the following: 35,642; 1,763,208; 41,574; 34,825; 17,824; 34,176.

Find the following products and check:

5.	56.74	8.	70.48	11.	765.75	14.	345.19
	5.68		8.79		34.56		57.84

6.	74.38	9.	64.08	12.	495.06	15.	417.38
	6.75		7.45		28.34		517.48

7.	68.59	10.	35.74	13.	305.67	16.	571.84
	34.15		42.07		95.07		304.52

17. Find the cost of 752 doz. eggs at 39 ¢ per dozen.

18. How much will be paid to 179 laborers for an 8-hr. day at 30 ¢ per hour ?

19. Find the cost of 267 lb. poultry at 28 ¢ per pound.

20. What will be the fare for 3 people going 587 mi. and return, if the fare is 3 ¢ per mile ?

21. What will be the fare for a football team which goes to play another team 149 mi. away, if 18 players are taken on the trip and the fare is 3 ¢ per mile ?

22. A farmer estimated that it cost him $ 11.80 per acre to raise and harvest his corn. What did it cost him to raise 85 acres of corn ?

23. His corn, which sold at $ 1.19 per bushel, averaged 37 bu. per acre. What were the returns from his corn ?

24. From Exs. 22 and 23 find the profits per acre planted to corn.

25. What is the cost of a 3-lb. duck at 28 ¢ per pound ? What change will be received from a $ 2 bill ? If you were cashier, how would you give out the change ?

26. If the dimensions of a rectangular basement are stated in yards, show that the number of cubic yards of dirt removed will be stated by the formula

$$N = LWD.$$

27. Find the volume of a basement in cubic yards that is 45′ by 30′ by 8′; that is 43′ by 28′ by 6′.

28. What is the cost of a 5-lb. roast at 36 ¢ per pound? How would you give the change from a $ 5 bill?

18. Division.—Use short division with divisors of one digit and long division with divisors of two or more digits. If the second digit of the divisor is small, as in 529, the first digit can be used as a trial divisor; if the second digit is large, as in 684, the first digit should be increased by 1 for trial divisor.

Division with decimals may be simplified by multiplying dividend and divisor by such a power of 10 as will remove the decimal point from the divisor.

19. Checking Division.—It is customary to multiply the divisor by the quotient and add to this the remainder to check division. A better method is to make the quotient the multiplicand, for then the multiplications will be different and mistakes of multiplication while dividing are not retained in the mind and thus repeated.

A still better way is based on excesses of 9's. In the equation connecting the numbers in division,

$$D = d \times q + r,$$

use the excesses of each in the place of the numbers themselves. If the division is correct, the excesses will still give a true equation.

Suppose that 8711 divided by 25 gives a quotient of 348 and a remainder of 11. Then

$$8711 = 25 \times 348 + 11.$$

By excesses: excess of 25 = 7, excess of 348 = 6, excess of 11 = 2. $7 \times 6 + 2 = 44$, which has an excess of 8. What is the excess of 8711? What is the conclusion? Can work that checks contain errors?

EXERCISES

1. State the law governing the placing of the decimal point in division. How can this be simplified through the suggestion given in Art. 18 ?

Carry out the following divisions and check:

2. $678 \div 51$
3. $546 \div 42$
4. $38.62 \div 28$
5. $456.4 \div 39$
6. $654.23 \div 5.9$
7. $504.73 \div 6.3$

8. $257.32 \div 6.8$
9. $274.19 \div 5.7$
10. $234.17 \div 234.5$
11. $3452.46 \div 371.5$
12. $345.07 \div 72.5$
13. $130.45 \div 39.43$

14. Find the cost of 6450 lb. coal at $ 4.50 per ton. How much is this per 1000 lb. ?

15. Find the cost of 8560 lb. coal at $ 3.75 per ton.

16. A bushel of shelled corn weighs 56 lb. Find the cost of 840 lb. shelled corn at $ 1.17 per bushel.

17. A bushel of wheat weighs 60 lb. Find the value of 3820 lb. wheat at $ 1.98 per bushel.

18. If 833 bu. of wheat are harvested from 45 acres, what is the yield per acre ?

19. A farmer picked 1540 bu. of corn from a field of 65 acres. What was the yield per acre ?

20. What is the gain or loss in buying a house for $ 2850 and selling it for $ 2325 ? What part of the purchase price is gained or lost ?

21. A train went 468.6 mi. in 16.25 hr. What was its hourly rate of speed ?

22. At the rate of 28 mi. per hour, how far will a train go in 8.5 hr. ? in 18 hr. ?

23. How long will it take a train going at the rate of 30 *mi. per hour* to go 450 mi. ? 378.6 mi. ?

24. What will be the cost of the freight for the following ?

5640 lb. at 29 ¢ per cwt.

3850 lb. at 35 ¢ per cwt.

3940 lb. at 47 ¢ per cwt.

25. Find the cost at $ 6.45 per cwt. of three purchases of sugar of 450 lb., 625 lb., and 375 lb.

26. What will be the freight on 6970 bu. of corn at the rate of $ 5.35 per 100 bu. ?

27. What will 4560 bd. ft. of lumber cost at $ 56 per 1000 bd. ft. ? at $ 72 per M ?

28. What will 7650 bd. ft. of lumber cost at $ 8.50 per 100 bd. ft. ?

29. A wagon of ear corn weighed 3050 lb. and the wagon empty weighed 970 lb. How many bushels of corn did the wagon contain at 70 lb. to the bushel ?

30. State by an equation the number of bushels, *B*, contained in a load weighing *W* pounds and the wagon empty *w* pounds, for wheat, oats, and barley.

31. A wagon full of coal weighs 5350 lb. and the wagon weighs 1430 lb. Find the cost of the coal at $ 5.45 per ton.

32. A wagon of coal weighed 3580 lb. and the wagon empty weighed 1390 lb. Find the cost of the coal at $ 6.75 per ton.

33. How many tons of coal, *T*, in a load weighing *W* pounds and the wagon weighing *w* ? Express its price at $ 8.00 per ton.

34. What is the cost of a 9-lb. turkey at 36 ¢ per pound ? How would you make the change, if paid with a $ 5 bill ?

35. Find the cost of the following: 6 cans corn at 19 ¢; 4 cans peaches at 43 ¢; 3 loaves of bread at 10 ¢; 4 packages breakfast food at 14 ¢. How would you make change if tendered a $ 5 bill ?

II

SHORT CUTS IN COMPUTATIONS

20. Short Cuts.—The value of short cuts in simplifying computations and in saving time cannot be overestimated. At first short cuts will, of course, take longer time than the known processes, just as writing shorthand takes more time at first than the known longhand. Practice and continual watchfulness for short cuts will soon show their value.*

21. Short Cuts in Addition.—To add 8 increase ten's digit by 1 and decrease unit's digit by 2. Thus,

$$45 + 8 = (40 + 10) + (5 - 2) = 53.$$

To add 9 increase ten's digit by 1 and decrease unit's by 1. Thus,

$$48 + 9 = (40 + 10) + (8 - 1) = 57.$$

This principle can also be applied to adding a one digit number to any number ending in 8 or 9, as $5 + 58$, or $7 + 39$. Formulate this short cut fully.

To add two numbers of two digits each, first find the sum of the tens' digits, to this 1 must be added if the sum of the units' digits is 10 or more, to this annex the last digit of the sum of units' digits. Thus, in $52 + 24$ we have $5 + 2 = 7$ and $2 + 4 = 6$, or 76; in $47 + 35$ we have $4 + 3 + 1 = 8$ and $7 + 5 = 12$, or 82. Explain fully.

* **Note to Teacher.**—The short cuts are placed together in one chapter for convenience of reference. In order, however, to show the students the value of short cuts they must be taken slowly and with the other work of the book at the rate of one or two articles per week. As soon as a short cut has been learned, require its application whenever occasion arises in the exercises of this book as well as in other computations.

EXERCISES

Carry out the following additions quickly:

1.	46	53	48	24	15	63	47	25
	9	8	8	9	9	8	9	8

2.	43	52	47	74	73	35	24	39
	8	9	8	8	9	9	8	9

3.	29	38	69	58	69 ·	78	28	39
	6	5	7	4	5	7	6	8

4.	37	86	75	39	68	39	58	79
	9	8	9	6	6	7	5	4

5. Let the teacher give you other lists like this to practise upon.

6.	45	64	72	84	93	39	48	84
	23	32	46	53	13	21	42	56

7.	35	74	47	38	83	24	37	73
	16	17	34	13	18	18	35	58

8.	64	38	72	95	51	94	46	52
	32	45	79	61	15	49	67	89

9.	65	27	38	96	87	93	67	53
	48	65	57	23	56	38	85	68

10. Let the teacher give you other lists like this for practise.

11. Give some illustrations in which it would be convenient to add 2 numbers of 2 digits each without the use of pencil and paper. Let the class carry out these additions.

22. Further Short Cuts in Addition.—Additions can often be simplified by adding and subtracting the same number. Thus,

$$57 + 34 = (57 + 3) + (34 - 3) = 60 + 31 = ?$$

One number may sometimes be broken up into the sum of two other numbers. Thus,

$$35 + 27 = 35 + 25 + 2 = 60 + 2 = ?$$

Three or more numbers may often be rearranged advantageously. Thus,

$$47 + 35 + 23 = 47 + 23 + 35 = 70 + 35 = ?$$

EXERCISES

Carry out the following without using pencil and paper:

1. $68 + 26$	**4.** $27 + 15$	**7.** $47 + 25 + 13$
2. $57 + 38$	**5.** $43 + 19$	**8.** $25 + 19 + 15$
3. $39 + 43$	**6.** $38 + 24$	**9.** $64 + 37 + 34$

10. Let the teacher suggest other exercises for practice.

11. A grocery bill contained the following amounts: 37 ¢, 25 ¢, and 23 ¢. Give the total amount.

12. John's sales of magazines for 4 weeks were 17, 15, 13, and 12. How many magazines did he sell during the 4 weeks?

13. Henry earned 35 ¢ one day and 47 ¢ the next day. How much did he earn during the two days?

14. Mary put into her savings-bank 47 ¢, 25 ¢, and 33 ¢ for each of 3 weeks. How much did she put into her savings-bank during these 3 weeks?

15. $3a[2a - 5b] = ?$ **19.** $(2m + 3n)(3m^2 - 5mn + 6n^2) = ?$

16. $7r(3r^2 + 4r) = ?$ **20.** $[5s - 7q][4s^3 + 3s^2q - sq^2] = ?$

17. $5g\{5g^3 - 7g^2\} = ?$ **21.** $(5k - 5v)(7k^2 - 6kv + 9v^2) = ?$

18. $7x[3x^2 - 4x^3] = ?$ **22.** $\{7u + 5y\}\{8u^2 + 7uy - 6y^2\} = ?$

23. Short Cuts in Subtraction.—To subtract from a power of 10, as 100 or 1000, think of it as $90 + 10$ or $990 + 10$. Begin at the left and subtract each digit from 9, except unit's digit, which is subtracted from 10.

$$1000 = 99^{10}$$
$$563 = \underline{563}$$
$$437$$

In subtracting numbers of two digits, if the unit's digit of the subtrahend is the larger, add enough to it to make zero and then add an equal amount to the minuend. Why will this give the correct difference?

$$43 - 27 = (43 + 3) - (27 + 3) = 46 - 30 = ?$$

EXERCISES

Subtract the following rapidly:

1.	56	73	64	82	34	52	16	29	32
	28	35	21	35	16	36	7	17	15

2.	43	63	32	75	84	61	52	64	73
	28	36	15	47	38	43	26	38	69

3.	1000	100	1000	100	1000	100	1000
	546	38	273	29	375	37	458

4.	563000	248000	360000	382000	524000
	563	247	142452	75764	316527

5. A lady paid 25 ¢ a can for salmon that she bought two years ago for 18 ¢ a can. How much more did she pay for it this year?

6. What change will be received from a $ 10 bill if the purchase was $ 4.35? $ 6.35? $ 7.85?

7. What change will be received from a $ 5 bill if the purchase was $ 2.65? $ 3.26? $ 2.97?

24. Short Cuts in Multiplication and Division.—While several short cuts arise in division, it is in multiplication that they abound. Often seemingly difficult multiplications can easily be reduced to mental processes or at least be greatly simplified.

25. Multiplication and Division by a Power of 10.—State the rule for multiplying by 10; by 100; by 1000; by any power of 10. State the rule for dividing by 10; by 100; by 1000; by any power of 10.

26. Buying and Selling by the Hundred or the Thousand.—Point off the proper number of places and find the product of this number and the given price.

27. Buying and Selling by the Ton.—First divide the amount in pounds sold by 1000 and then by 2; or divide the price by 2, which gives the price per 1000.

<div align="center">EXERCISES</div>

Give at sight the following five exercises:

1. 34.56×100 5.67×100 34.07×10^3 70.34×10^3

2. 3.005×10^2 29.005×10^3 4.5073×10^2 0.00453×10^3

3. 0.0305×10^4 34.05×10^4 34.0025×10^2 1.0035×10^3

4. $34.56 \div 100$ $30.056 \div 10^2$ $9.0035 \div 10^2$ $340.23 \div 10$

5. $360.25 \div 10^3$ $340.16 \div 10^2$ $2025.5 \div 10^2$ $3045.5 \div 10^2$

6. How many tons are there in 3400 lb. ? 4800 lb. ? 2460 lb. ? 5640 lb. ? 3280 lb. ? 2640 lb. ? 1960 lb. ?

7. Find the cost of 4820 lb. soft coal at $ 5.25 per ton.

8. Find the cost of 3710 lb. hard coal at $ 9.50 per ton.

9. Find the cost of 5960 ft. lumber at $ 60 per M.

28. Changing Order of Multipliers.—The product of $25 \times 719 \times 4$ can be found with the numbers arranged in six different ways or orders. What are these? Some of these make the work a simple mental operation; as, $25 \times 4 \times 719$. Carry this out. Such rearranging of the order in multiplications and divisions greatly simplifies the work.

<center>EXERCISES</center>

Carry out without pencil and paper as far as possible:

1. $4 \times 319 \times 25$	**5.** $4 \times 9 \times 75$	**9.** $7 \times 4 \times 25$
2. $6 \times 37 \times 5$	**6.** $8 \times 37 \times 5$	**10.** $75 \times 32 \div 25$
3. $4 \times 76 \times 50$	**7.** $5 \times 79 \times 2$	**11.** $64 \times 25 \div 16$
4. $2 \times 59 \times 5$	**8.** $2 \times 250 \times 7$	**12.** $12 \div 7 \times 35$

13. Suggest five multiplications that can be simplified by this process.

29. Multiplying by 11, 22, etc.—To multiply by 11, multiply by 10 and add to this the given number. In practice, write unit's digit of the number multiplied for unit's digit in the product; add unit's and ten's digit for ten's digit in the product; same for the other digits. Explain. To multiply by 22, 33, etc., multiply by 11 and then each digit by 2, 3, etc., before writing the product.

$$\begin{array}{r} 752 \\ 11 \\ \hline 8272 = 8272 \end{array} \qquad \begin{array}{r} 7520 \\ 752 \\ \hline 8272 \end{array} \qquad \begin{array}{r} 342 \\ 22 \\ \hline 7524 = 7524 \end{array} \qquad \begin{array}{r} 6840 \\ 684 \\ \hline 7524 \end{array}$$

<center>EXERCISES</center>

Carry out mentally as far as possible:

1. 352×11	**5.** 425×22	**9.** 413×22
2. 564×11	**6.** 417×11	**10.** 324×33
3. 809×11	**7.** 506×11	**11.** 725×11
4. 213×22	**8.** 758×11	**12.** 864×11

30. Multiplying by Sums or Differences.—Many simplifications are based upon the principles,

$$a(b + c) = ab + ac,$$

and
$$a(b - c) = ab - ac.$$

Thus, $15 \times 17 = 15(15 + 2) = 15^2 + 2 \times 15$, which is a mental operation, as 15^2 is 225 and 2×15 is 30, while their sum is 255. Similarly,

. $15 \times 13 = 15(15 - 2) = 15^2 - 2 \times 15 = 225 - 30 = ?$

31. Multiplier Near a Power of 10.
—Multiplying by 97, 998, etc., is another application of the principle in Art. 30. $97 = 100 - 3$; $998 = 1000 - 2$; etc. Study out carefully the multiplication here given.

$$
\begin{array}{rl}
245 & \\
98 &= 100 - 2 \\
\hline
24500 &= 100 \times 245 \\
490 &= 2 \times 245 \\
\hline
24010 &= \text{difference}
\end{array}
$$

EXERCISES

Use pencil and paper as little as possible.

1. 9×45	**3.** 19×15	**5.** 19×60	**7.** 42×15
2. 9×24	**4.** 28×30	**6.** 23×15	**8.** 13×25

9. Suggest at least five multiplications that can be simplified by the use of the principle in Art. 30.

10. 215×97	**12.** 325×997	**14.** 998×534
11. 627×98	**13.** 647×996	**15.** 997×576

16. A meter is 1.09 yd. How many yards long are 98 m. ?

17. When a draft payable in gold in France costs 19.50 ¢ per franc in the United States, what will be the cost of a draft of 500 francs ? of 4000 francs ? of 8500 francs ?

18. What should I pay a man working for me 9 hr. at 35 ¢ per hour ?

19. How many square feet are there in the floor of a room 15 ft. by 18 ft. ? 12 ft. by 16 ft. ? 19 ft. by 24 ft. ?

32. Product of Sum and Difference.—A basis of another simplification is

$$(a + b)(a - b) = a^2 - b^2.$$

Thus, $48 \times 52 = (50 - 2)(50 + 2) = 50^2 - 2^2$
$$= 2500 - 4 = ?$$

EXERCISES

Use pencil and paper as little as possible.

1. 38×42 3. 54×46 5. 402×398 7. 227×233
2. 26×34 4. 77×83 6. 245×255 8. 240×260

9. Find the area of a floor that is 18 ft. by 22 ft.; 28 ft. by 32 ft.; 17 ft. by 23 ft.

10. If 25 lb. of milk make 1 lb. of butter, how many pounds of milk will it take to make 15 lb. of butter?

11. Find the cost of 1430 ft. flooring at $ 85 per M.

12. By paying cash a merchant pays only .98 of a bill of $ 345 for goods. How much cash does he pay?

13. How much cash is needed to pay a bill of $ 785.25, if the cash terms are .97 of the bill?

14. At 45 ¢ per hour, what must be paid a man for working 9 hr.? 19 hr.? 12 hr.? 33 hr.?

15. Find the cost of 18 qt. of milk at 12 ¢ per quart.

16. At the rate of 42 mi. per hour how far will a train go in 11 hr.?

17. What will be the cost of 305 cwt. of freight at $ 2.95 per C.?

18. Find the cost of 16 yd. of cloth at 24 ¢ per yard; of 14 yd. at 22 ¢ per yard.

19. In finding the interest on $ 5500 at 6.5 per cent, it is necessary to multiply 5500 by .065. Carry this out mentally; be careful about the decimal point in the final result.

20. Suggest at least two possibilities for use of the principle at the top of the page.

33. Factors.—Many very useful simplifications in multiplication and division depend upon factors of numbers. Give four factors of 30.

34. Divisibility by 2, 4, and 8.—What numbers are divisible by 2? Illustrate.

Any number of 3 digits or more can be represented by $100a + 10b + c$. What are c, b, and a in 375? in 4562? $(100a + 10b + c) \div 4 = 25a + (10b + c) \div 4$. Explain. This gives a whole number as quotient if $(10b + c) \div 4$ is a whole number. Hence, a number is divisible by 4 if the part formed by the last two digits is divisible by 4. Test for 2316, 3215, and 3408.

This also applies to divisibility by 25. Carry this out.

If the part of a number formed by the last three digits is divisible by 8, the number is divisible by 8. Any number containing at least three digits is of the form $1000a + 100b + 10c + d$. Complete as in the above equation, where 8 is used with the last three parts as 4 was used above. This test also applies to divisibility by 125. Carry this out.

35. Divisibility by 5 and 10.—State the tests for divisibility by 5; by 10; by powers of 10.

<div align="center">EXERCISES</div>

Test each for divisibility by 2, 4, 8, 5, and 10:

1. 35	24	348	560	712	9345
2. 48	75	211	749	624	3168
3. 96	30	720	456	824	3264
4. 73	60	325	604	936	7104
5. 90	83	427	728	825	6412

36. Divisibility by 3 and by 9.—If a number has no excesses of 9's—see Art. 10—it is divisible by 9; that is, if the sum of its digits is divisible by 9, the number is·divisible by 9. Similarly, if the sum of the digits of a number is divisible by 3, the number is divisible by 3. Apply the tests to the numbers in Exs. 1, 2, and 3 on the preceding page.

37. Combining Tests of Divisibility.—An even number is divisible by 6 or 18 if the sum of its digits is divisible by 3 or 9 respectively. Explain fully. Apply to 168, 3654, and 354.

A number ending in 5 or 0 is divisible by 15 or 45 if the sum of its digits is divisible by 3 or 9 respectively. How ? Apply to 345, 6345, 135, 2610, and 1410.

EXERCISES

1. When is a number divisible by 4 ? by 3 ? Hence, when is a number divisible by 12 ?

2. When is a number divisible by 20 ? by 75 ?

3. When is a number divisible by 30 ? by 50 ?

4. Test the numbers in Exs. 3, 4, and 5, page 24, for divisibility by 6, 18, 15, 12, 20, 50, and 75.

38. Prime Factors.—Numbers can often be expressed by their factors in several different ways. Thus,

$$30 = 3 \times 10 = 5 \times 6 = 2 \times 15 = 2 \times 3 \times 5.$$

Numbers that cannot be factored are called **prime numbers.** As the 2, 3, and 5 cannot be factored, they are the **prime factors** of 30.

Find the prime factors of the numbers in the first four columns of the exercises on page 24.

39. Separating Multipliers into Factors.—Several short cuts depend upon factors. One of the most useful is the factoring of one multiplier so as to give simpler arrangements for multiplying. Thus,

$$56 \times 25 = 7 \times 8 \times 25 = 7 \times 200 = ?$$

Multiplication by a number in which one part is a multiple of the other is an extension of the above idea. Thus, to multiply by 248 is the same as to multiply first by 8 and then by 30 times this result because $240 = 30 \times 8$. Explain fully.

$$
\begin{array}{l}
213 \\
\underline{248} \\
\overline{1704} = 213 \times 8 \\
51120 = 1704 \times 30 \\
\overline{52824}
\end{array}
$$

EXERCISES

Use pencil and paper as little as possible.

1. 9×27	16×45	356×248	435×126
2. 8×45	24×15	488×738	327×219
3. 6×85	18×65	714×234	728×536
4. 8×35	42×15	517×426	345×998
5. 6×75	12×35	273×529	754×355
6. 4×95	55×16	287×546	997×456

7. What is the value of 160 A. land at $ 22.50 per acre ?

8. What will a wholesale merchant pay for 317 crates of berries at $ 4.26 per crate ?

9. What is the area of a floor 18 ft. by 27 ft. ? 15 ft. by 18 ft. ? 25 ft. by 28 ft. ?

10. What will a merchant pay for 124 chairs at $ 6.75 each ?

11. What is the cost of 80 A. land at $ 11.25 per acre ?

12. What is the cost of 12500 bricks at $ 8.40 per 1000 ?

13. What is the cost of 45820 lb. of coal at $ 3.55 per *ton ?*

40. Factors in Division.—Division can often be simplified by casting out common factors from both dividend and divisor. If the division is carried out in decimals, the decimal part will be exact. Thus, $248 \div 36 = 62 \div 9 = 6.9$. If the division is not carried out to decimals, any remainder must be multiplied by the factor that was excluded. Why? Apply to $248 \div 36$.

<div align="center">EXERCISES</div>

Carry out the following in the simplest manner to two decimal places:

1.	$548 \div 20$	**5.**	$784 \div 28$	**9.**	$512 \div 36$
2.	$252 \div 42$	**6.**	$225 \div 18$	**10.**	$1236 \div 30$
3.	$1625 \div 45$	**7.**	$625 \div 75$	**11.**	$2345 \div 125$
4.	$513 \div 12$	**8.**	$936 \div 36$	**12.**	$7895 \div 30$

41. Multiplying by 5, 25, 125.—Since $5 = 10 \div 2$, multiplying by 5 is the same as multiplying by 10 and then dividing by 2. Multiplying by 10 is merely annexing a zero, while dividing by 2 is simpler than multiplying by 5. Thus,

$$247 \times 5 = 2470 \div 2 = 1235.$$

Since $25 = 100 \div 4$, how would you multiply by 25?
Since $125 = 1000 \div 8$, how would you multiply by 125?

Show that $a \times 5 = \dfrac{10a}{2}$; $a \times 25 = \dfrac{100a}{4}$; $a \times 125 = \dfrac{1000a}{8}$.

In practice, merely **think** the zeros annexed and divide.

<div align="center">EXERCISES</div>

Use pencil and paper as little as possible.

1.	24×5	324×25	32×125	4608×25
2.	57×5	506×25	24×125	3420×25
3.	64×5	518×25	56×125	7284×25
4.	78×5	907×25	29×125	6806×25
5.	67×5	645×25	34×125	3903×25

42. Dividing by 5, 25, 125.—As division is the reverse of multiplication, to divide by 5, multiply by 2 and divide by 10, that is, point off one place.   Thus,

$$374 \div 5 = 374 \times 2 \div 10 = 748 \div 10 = 74.8.$$

According to this principle, how can we divide by 25 ? by 125 ? Show that

$$a \div 5 = \frac{2a}{10}; \ \ a \div 25 = \frac{4a}{100}; \ \ a \div 125 = \frac{8a}{1000}.$$

EXERCISES

Use pencil and paper as little as possible.

1. 235 ÷ 5	324 ÷ 25	2314 ÷ 125	216 × 25
2. 408 ÷ 5	765 ÷ 25	4305 ÷ 125	408 × 25
3. 415 ÷ 5	806 ÷ 25	6231 ÷ 125	417 × 98
4. 702 ÷ 5	971 ÷ 25	4056 ÷ 125	746 × 50
5. 637 ÷ 5	461 ÷ 25	8736 ÷ 125	325 × 19
6. 472 ÷ 5	537 ÷ 25	3729 ÷ 125	647 × 25

7. A junior high school class of 86 pupils attended a football game.   At 25 ¢ each what did they pay for all ?

8. A junior high school class decided to invest the money in its treasury, $ 17.28, in Thrift Stamps.   How many Thrift Stamps did the class buy ?

43. Aliquot Parts in Multiplication.—A number that divides another without a remainder is said to be an **aliquot part** of that other number.   Thus, 5 and 2 are the aliquot parts of 10.   The most useful aliquot parts of 100 are $12\frac{1}{2}$, $16\frac{2}{3}$, 25, $33\frac{1}{3}$, and 50.   While $66\frac{2}{3}$ and 75 are not aliquot parts of 100, they can be used in much the same manner.

To multiply by $12\frac{1}{2}$, annex two zeros and divide by 8. Why ?   Thus,

$$324 \times 12\frac{1}{2} = 32400 \div 8 = 4050.$$

Explain the use of the other aliquot parts of 100.

44. Extension of Aliquot Parts.—

In multiplying by such a number as 733⅓, note that 33⅓ is ⅓ of 100. Hence, multiply by 7⅓ **hundreds.** Why ?

$$
\begin{array}{r}
345 \\
733\frac{1}{3} \\
\hline
11500 = 33\frac{1}{3} \times 345 \\
241500 = 700 \times 345 \\
\hline
253000
\end{array}
$$

45. Aliquot Parts in Division.—Division is again treated as the reverse of multiplication. Hence, to divide by 16⅔, multiply by 6 and divide by 100, that is, point off two places after multiplying. Thus,

$$526 \div 16\frac{2}{3} = 526 \times 6 \div 100 = 3156 \div 100 = ?$$

Explain the use of the other aliquot parts of 100 in division.

EXERCISES

Use pencil and paper as little as possible.

1. 246 × 25	743 × 325	124 × 312½
2. 763 × 75	857 × 416⅔	547 × 933⅓
3. 642 ÷ 25	475 × 766⅔	907 × 475
4. 372 ÷ 12½	893 ÷ 66⅔	758 × 716⅔
5. 819 ÷ 33⅓	572 × 416⅔	548 × 733⅓
6. 156 ÷ 75	496 × 250	849 × 566⅔
7. 125 ÷ 12½	805 ÷ 250	709 × 733⅓

8. John owns a cow and sells milk at 12½ ¢ per quart. A family taking a quart each day pays him how much during February, 1919 ? during June ? during July ?

9. Mary deposits 25 ¢ each week in her savings-bank. How much does she deposit in a year ? in 26 wk. ?

10. Mabel is paid 16⅔ ¢ per hour for working. How many hours does she work for one dollar ? for $ 5 ? How much is she paid for working 23 hr. one week ?

46. Special Short Cuts.—In addition to the general short cuts which have just been studied, a few arising less frequently will be given. Any that may later be met frequently in connection with some activity or business should be learned thoroughly. They are great time-savers.

47. Multiplying in the 'Teens.—Any two numbers between 10 and 20 can be represented by $10 + a$ and $10 + b$. How? Their product will then be

$(10 + a)(10 + b) = 100 + 10(a + b) + ab$ How?

Applying to $14 \times 18 = 100 + 10(4 + 8) + 32$.

$$= 100 + 10(3 + 4 + 8) + 2 = 252.$$

Note that ab gives units and perhaps tens; $10(a + b)$ gives tens and perhaps hundreds, to which is always added 100. In practice, do not use formula, but **think** $4 \times 8 = 32$; write two units and carry three tens. 3 (carried) $+ 4 + 8 = 15$ tens; to this add ten tens, from 100, and write 25 before the two units.

48. Extension of Multiplying in the 'Teens.—The last principle can be extended to numbers a little above 100 or 1000.

$(100 + a)(100 + b) = 10,000 + 100(a + b) + ab$

$105 \times 116 = 10,000 + 100(5 + 16) + 5 \times 16$

$$= 12,180.$$

Note that ab gives units and tens, and perhaps hundreds; $100(a + b)$ gives hundreds and thousands, and perhaps 10,000, to which is always added 10,000.

EXERCISES

Carry out the following mentally:

1. 13×15 12×18 19×11 15×15 104×106

2. 17×13 19×15 18×15 13×18 107×103

49. Sum of Units' Digits 10 in 'Teens.—If $a + b = 10$,

$$(10 + a)(10 + b) = 100 + 10(a + b) + ab$$
$$= 100 + 100 + ab = 200 + ab.   \text{How?}$$

To apply, merely place 2 before the product of the units' digits; thus,

$$14 \times 16 = 200 + 4 \times 6 = 224.$$

50. Units' Digits Same.—Then we have,

$$(10a + b)(10c + b) = 100ac + 10b(a + c) + b^2.   \text{How?}$$
Hence, $74 \times 24 = 100(7 \times 2) + 10 \times 4(7 + 2) + 4^2$
$$= 1400 + 360 + 16 = 1776.$$

In practice, do not use the formula, but **think** $4^2 = 16$; write the 6 units and carry the 1 ten. Tens are 1 (carried) + $4(7 + 2) = 37$; write the 7 tens and carry 3. Hundreds are 3 (carried) + $7 \times 2 = 17$; write 17 before 76.

51. Both Units' Digits 5.—Then,

$$(10a + 5)(10b + 5) = 100ab + 50(a + b) + 25,   \cdot\text{How?}$$
$$= 100 \left(ab + \frac{a + b}{2} \right) + 25.   \text{How?}$$

Hence, $45 \times 65 = 100 \left(4 \times 6 + \frac{4 + 6}{2} \right) + 25$
$$= 100(24 + 5) + 25 = 2925.$$

In practice, add mentally the tens and divide by 2. To this quotient add the product of the tens and write this result with 25 after it, if tens' digits are both odd or both even, and 75 after this sum, if tens' digits are one even and one odd.

<div align="center">

EXERCISES

</div>

Carry out mentally:

1.	13×17	19×11	12×18	16×14	315×15
2.	35×25	13×23	15×45	16×36	415×85
3.	17×47	25×75	24×64	32×82	23×53

52. Tens' Digits Same.—The product is

$(10a + b)(10a + c) = 100a^2 + 10a(b + c) + bc.$ **How?**

Hence, $34 \times 37 = 100 \times 3^2 + 10 \times 3(4 + 7) + 4 \times 7$
$$= 900 + 330 + 28 = 1258.$$

In practice, **think** $4 \times 7 = 28$; write 8 units and carry 2 tens. The tens are 2 (carried) $+ 3(4 + 7) = 35$; write 5 tens and carry 3 hundreds. Hundreds are 3 (carried) $+ 3^2 = 12$.

53. Squaring a Number Ending in 5.—The form is

$$(10a + 5)^2 = 100a^2 + 2 \times 5 \times 10a + 25$$
$$= 100a^2 + 100a + 25$$
$$= 100a(a + 1) + 25. \qquad \textbf{How?}$$

Hence, $(75)^2 = 100 \times 7(7 + 1) + 25$
$$= 100 \times 7 \times 8 + 25 = 5625.$$

In practice, multiply mentally the tens' digit by tens' digit increased by one—7×8—and place 25 after this number.

<center>**EXERCISES**</center>

Use pencil and paper as little as possible.

1. 24×28 37×35 52×54 73×76 89×82 $(65)^2$

2. 56×52 48×43 53×57 16×18 27×29 $(35)^2$

3. 42×45 65×63 85×82 92×97 34×37 $(85)^2$

4. 28×21 38×32 56×51 74×71 63×65 $(45)^2$

5. Find the price of 23 yd. of cloth at 29 ¢ per yard.

6. What is the cost of 38 doz. eggs at 34 ¢ per dozen ?

7. What should be paid for 43 hr. labor at 45 ¢ per hour ? for 38 hr. labor at 35 ¢ per hour ?

8. How much should you receive for a load of 58 bu. oats at 52 ¢ per bushel ?

III

REVIEW FRACTIONS

54. Fractions.—A **fraction** is a part of an object or of a group of objects.

55. Denominator.—In the fraction ¾, the number below the line, 4, is called the **denominator** of the fraction. The denominator **names** the fraction; it tells into how many parts the whole number has been divided.

56. Numerator.—The number above the line is called the **numerator.** It is the **numbering** part of the fraction; it tells how many of the parts of the whole number are indicated.

57. Terms.—Numerator and denominator together are called **terms** of the fraction.

58. Proper Fractions.—**Proper fractions** have the denominator greater than the numerator; they are less than one, as ⅖.

59. Improper Fractions.—**Improper fractions** have the numerator equal to or greater than the denominator; they are one or more, as ⅚ or ⅞.

60. Mixed Numbers.—A number consisting of a whole number and a fraction is called a **mixed number**, as 4⅖.

EXERCISES

1. What is the effect on a fraction of increasing its numerator? increasing its denominator?

2. Which is greater, ¼ or ⅜? ⅕ or ₄⁄₁₀? ⅓ or ⅜?

3. What part of a dollar is 25 ¢? 20 ¢? 75 ¢? 36 ¢?

33

61. Reduction to Equivalent Fractions.—Which is more, $\frac{1}{3}$ or $\frac{2}{5}$ of a yard ? $\frac{1}{2}$ or $\frac{6}{10}$ of a dollar ? Multiplying or dividing both terms of a fraction by the same number does not change its value. Such changes are called **reductions,** and the fractions are said to be **equivalent.** Thus, $\frac{4}{6} = \frac{8}{12} = \frac{2}{3}$.

62. Reduction to Lowest Terms.—Reductions in which the terms are made as small as possible are referred to as **reductions to lowest terms.** Such are, $\frac{4}{12}$ to $\frac{1}{3}$, and $\frac{3}{15}$ to $\frac{1}{5}$. Suggest a fraction that is not in its lowest terms and reduce it.

In all operations giving a fraction in the final result, reduce the fraction to lowest terms.

EXERCISES

1. Change each of the following to two other equivalent fractions: $\frac{3}{4}$; $\frac{2}{5}$; $\frac{5}{6}$; $\frac{3}{12}$; $\frac{7}{8}$; $\frac{3}{8}$; $\frac{13}{18}$; $\frac{3}{8}$; $\frac{4}{7}$; $\frac{4}{24}$; $\frac{4}{7}$; $\frac{15}{18}$; $\frac{9}{24}$; $\frac{8}{32}$; $\frac{5}{75}$; $\frac{2}{9}$.

2. Reduce to their lowest terms: $\frac{4}{8}$; $\frac{12}{18}$; $\frac{8}{12}$; $\frac{33}{66}$; $\frac{27}{36}$; $\frac{15}{18}$; $\frac{45}{9}$; $\frac{33}{22}$; $\frac{66}{88}$; $\frac{42}{7}$; $\frac{44}{72}$; $\frac{48}{28}$; $\frac{48}{66}$; $\frac{22}{48}$; $\frac{24}{81}$; $\frac{48}{88}$; $\frac{88}{88}$.

63. Addition and Subtraction.—Fractions having the same denominators can be added and subtracted just as other **named numbers.** To add or to subtract fractions not having the same denominators, first reduce them to equivalent fractions with the same denominators.
Thus, $\frac{3}{5} + \frac{1}{4} = \frac{12}{20} + \frac{5}{20} = \frac{17}{20}$.

EXERCISES

Carry out the following operations:

1. $\frac{2}{3} + \frac{3}{5}$ **4.** $\frac{2}{4} + \frac{5}{6} - \frac{1}{8}$ **7.** $\frac{5}{21} + \frac{3}{14} + \frac{1}{6}$

2. $\frac{7}{8} + \frac{3}{4}$ **5.** $\frac{1}{7} - \frac{3}{14} + \frac{5}{6}$ **8.** $\frac{3}{4} - \frac{3}{8} - \frac{1}{16}$

3. $\frac{1}{5} - \frac{2}{15}$ **6.** $\frac{2}{15} + \frac{3}{10} + \frac{5}{6}$ **9.** $\frac{5}{8} + \frac{3}{16} - \frac{7}{32}$

10. $\frac{5}{6} + \frac{7}{8}$ **13.** $\frac{5}{12} + \frac{7}{4} - \frac{2}{8}$ **16.** $\frac{1}{2} + \frac{1}{3} - \frac{5}{6}$

11. $\frac{7}{8} - \frac{1}{4}$ **14.** $1\frac{1}{6} - \frac{1}{5} - \frac{1}{3}$ **17.** $\frac{3}{8} - \frac{8}{16} + \frac{1}{2}$

12. $\frac{3}{4} - \frac{2}{3}$ **15.** $\frac{5}{6} + \frac{3}{4} + \frac{5}{8}$ **18.** $\frac{5}{6} - \frac{2}{3} + \frac{3}{4}$

19. Write the fraction having a numerator $4a$ and denominator $3c$; and another having a numerator $2c$ and denominator $5a$. Add these two fractions.

20. Find the cost of one article if n of them cost \$ d; if N of them cost \$ D. Add these two fractions to find the cost of one each of the two classes of articles.

21. The area and width of a rectangle are A and W; and of another a and w. Express the length of each. Add these two fractions to express the sum of their lengths.

64. Mixed Numbers to Improper Fractions.—A mixed number is really a whole number plus a fraction. Thus, $2\frac{3}{4} = 2 + \frac{3}{4} = \frac{8}{4} + \frac{3}{4} = \frac{11}{4}$. In practice, merely multiply the whole number by the denominator, add in the numerator, and place this over the denominator.

65. Improper Fractions to Mixed Numbers.—An improper fraction can be changed into the sum of two fractions, one of which can be reduced to a whole number. Thus, $\frac{34}{9} = \frac{27}{9} + \frac{7}{9} = 3 + \frac{7}{9} = 3\frac{7}{9}$. In practice, divide the numerator by the denominator; the quotient is the whole number and the remainder is the numerator of a new fraction with the old denominator.

EXERCISES

1. Change the following to improper fractions: $3\frac{1}{5}$; $2\frac{1}{5}$; $5\frac{1}{2}$; $3\frac{4}{5}$; $1\frac{2}{3}$; $4\frac{2}{3}$; $7\frac{2}{5}$; $6\frac{2}{7}$; $5\frac{3}{4}$; $12\frac{1}{4}$; $6\frac{3}{5}$; $7\frac{1}{5}$; $8\frac{3}{10}$; $4\frac{3}{5}$.

2. Change the following to whole or mixed numbers: $\frac{34}{5}$; $\frac{28}{7}$; $\frac{32}{26}$; $\frac{64}{8}$; $\frac{39}{4}$; $\frac{35}{7}$; $\frac{23}{4}$; $\frac{28}{8}$; $\frac{51}{17}$; $\frac{63}{8}$; $\frac{9}{5}$.

66. Adding Mixed Numbers.—First add the
fractions and reduce their sum to a mixed number
if possible. Place the whole number of this mixed
number above the other whole numbers and add the
whole numbers.

$$\begin{array}{r} 1 \\ 12\frac{3}{4} \\ 7\frac{1}{2} \\ 5\frac{1}{3} \\ \hline 25\frac{7}{12} \end{array}$$

Add the following:

1.	$25\frac{3}{4}$	3.	$18\frac{1}{2}$	5.	$5\frac{1}{6}$	7.	$11\frac{2}{3}$	9.	$8\frac{1}{6}$
	$8\frac{1}{6}$		$7\frac{1}{3}$		$12\frac{2}{5}$		$7\frac{3}{4}$		$17\frac{3}{5}$
	$7\frac{2}{3}$		$4\frac{3}{4}$		$6\frac{1}{3}$		$25\frac{1}{3}$		$16\frac{1}{2}$

2.	$22\frac{3}{8}$	4.	$8\frac{1}{7}$	6.	$27\frac{1}{6}$	8.	$27\frac{2}{3}$	10.	$7\frac{1}{5}$
	$6\frac{1}{4}$		$3\frac{1}{6}$		$8\frac{5}{6}$		$6\frac{1}{5}$		$16\frac{3}{4}$
	$12\frac{1}{2}$		$19\frac{2}{3}$		$12\frac{2}{3}$		$12\frac{1}{3}$		$22\frac{1}{2}$

67. Subtracting Mixed Numbers.—Subtract the frac-
tions and the whole numbers separately. If
the fraction in the subtrahend is larger than
that in the minuend, as in the example shown,
take 1 from the whole number of the minuend
and add to the fraction of the minuend.

$$\begin{array}{rcr} 14\frac{1}{4} & & 13\frac{5}{4} \\ 7\frac{1}{2} & & 7\frac{1}{4} \\ \hline 6\frac{3}{4} & = & 6\frac{3}{4} \end{array}$$

Carry out the following subtractions:

| 1. | $18\frac{3}{4}$ | 3. | $11\frac{5}{8}$ | 5. | $14\frac{7}{8}$ | 7. | $27\frac{1}{2}$ | 9. | $19\frac{1}{3}$ |
| | $7\frac{1}{2}$ | | $8\frac{2}{3}$ | | $6\frac{3}{4}$ | | $16\frac{3}{4}$ | | $8\frac{1}{2}$ |

| 2. | $35\frac{5}{6}$ | 4. | $27\frac{2}{5}$ | 6. | $17\frac{1}{6}$ | 8. | $28\frac{1}{4}$ | 10. | $27\frac{3}{4}$ |
| | $26\frac{1}{3}$ | | $19\frac{3}{4}$ | | $8\frac{2}{3}$ | | $16\frac{1}{2}$ | | $19\frac{5}{6}$ |

68. Adding and Subtracting by Cross Multiplication.—
With two fractions, multiply each numerator by the de-
nominator of the other fraction, and add or subtract for the

new numerator; then multiply the denominators together for the new denominator. Thus,

$$\frac{2}{3} + \frac{3}{4} = \frac{(2 \times 4) + (3 \times 3)}{3 \times 4} = \frac{17}{12} = 1\tfrac{5}{12};$$

$$\frac{2}{3} - \frac{3}{5} = \frac{(2 \times 5) - (3 \times 3)}{3 \times 5} = \frac{1}{15}.$$

69. Adding and Subtracting Unit Fractions.—If the numerators are 1, add or subtract the denominators for the new numerator and multiply the denominators together for the new denominator. Thus,

$$\frac{1}{2} - \frac{1}{3} = \frac{3 - 2}{3 \times 2} = \frac{1}{6}; \quad \frac{1}{8} + \frac{1}{5} = \frac{5 + 8}{8 \times 5} = \frac{13}{40}.$$

EXERCISES

Carry out mentally the following:

1. $\frac{1}{3} + \frac{1}{4}$	**5.** $\frac{1}{2} - \frac{1}{3}$	**9.** $\frac{1}{5} + \frac{1}{8}$	**13.** $\frac{2}{3} + \frac{3}{4}$
2. $\frac{1}{5} + \frac{1}{6}$	**6.** $\frac{1}{2} - \frac{1}{6}$	**10.** $\frac{1}{4} - \frac{1}{9}$	**14.** $\frac{3}{5} - \frac{2}{7}$
3. $\frac{1}{8} + \frac{1}{3}$	**7.** $\frac{1}{6} - \frac{1}{8}$	**11.** $\frac{3}{4} - \frac{1}{5}$	**15.** $\frac{4}{5} + \frac{2}{3}$
4. $\frac{1}{10} + \frac{1}{5}$	**8.** $\frac{1}{4} - \frac{1}{5}$	**12.** $\frac{1}{4} - \frac{1}{8}$	**16.** $\frac{3}{4} - \frac{2}{5}$

17. One day a grain dealer paid $89\tfrac{3}{4}$ ¢ per bushel for corn and the next day $93\tfrac{1}{3}$ ¢. What was the advance?

18. A grain dealer paid \$ $1.97\tfrac{3}{8}$ per bushel for wheat one day and \$ $1.96\tfrac{3}{4}$ the next day. How much more did he pay per bushel for wheat one day than the other day?

19. A merchant purchased the following pieces of velvet: $8\tfrac{3}{4}$ yd.; $16\tfrac{3}{8}$ yd.; $5\tfrac{7}{8}$ yd.; $11\tfrac{1}{4}$ yd. How many yards in all did he purchase?

20. It is estimated that a family with a yearly income of \$ 1500 should divide this into the following parts: $\frac{1}{5}$ for house rent; $\frac{1}{3}$ for food; $\frac{1}{6}$ for clothing; $\frac{1}{5}$ for other expenses; and the remainder for savings. What fractional part of the income will be for savings? how many dollars for savings?

70. Multiplying a Fraction by an Integer.—To multiply a fraction by an integer is to multiply the numerator—the numbering part of the fraction—by the integer. In practice, look for common factors between the denominator and the integer multiplier. Thus,

$$\frac{5}{\underset{3}{\cancel{12}}} \times \cancel{4} = ?$$

71. Multiplying Fractions by Fractions.—To multiply by 2 means to take 2 of the number multiplied. To multiply by $\frac{2}{3}$ means to take $\frac{2}{3}$ of that number; that is, take 2 times $\frac{1}{3}$ of the number. Hence, to multiply fractions by fractions multiply the numerators together for a new numerator and the denominators for a new denominator. In practice, look for common factors between numerators and denominators. Thus,

$$\frac{\cancel{5}}{\underset{3}{\cancel{6}}} \times \frac{4}{\underset{7}{\cancel{35}}} \times \frac{\overset{2}{\cancel{14}}}{9} = ?$$

Time can be saved by cancelling in the following manner: cancel 5, as it divides 35; cancel 35, but do not write the 7; cancel 14, as 7 divides it, but do not write the 2; cancel 6, as 2 divides it, and write the 3. Of course, this does not show the steps nor leave the numbers for checking the work.

EXERCISES

Carry out the following multiplications:

1. $\frac{2}{3} \times 12$
2. $\frac{8}{9} \times 15$
3. $\frac{15}{21} \times 14$
4. $\frac{7}{18} \times 27$
5. $\frac{7}{35} \times 15$
6. $\frac{7}{45} \times 18$

7. $\frac{3}{5} \times \frac{25}{27}$
8. $\frac{14}{15} \times \frac{25}{21}$
9. $\frac{10}{35} \times \frac{15}{12}$
10. $\frac{14}{15} \times \frac{3}{4}$
11. $\frac{24}{35} \times \frac{14}{4}$
12. $\frac{3}{8} \times \frac{24}{7}$

13. $\frac{2}{3} \times \frac{15}{16} \times \frac{8}{9}$
14. $\frac{5}{6} \times \frac{9}{35} \times \frac{23}{8}$
15. $\frac{3}{4} \times \frac{2}{3} \times \frac{4}{5}$
16. $\frac{7}{8} \times \frac{9}{35} \times \frac{5}{21}$
17. $\frac{4}{5} \times \frac{5}{6} \times \frac{6}{7}$
18. $\frac{5}{6} \times \frac{20}{21} \times \frac{48}{55}$

72. Multiplication of Mixed Numbers.—Reduce small mixed numbers to improper fractions first; multiply others directly, as shown below.

$$24\tfrac{3}{5}$$
$$15\tfrac{2}{7}$$

$$\frac{6}{35} \; is \; \tfrac{2}{7} \times \tfrac{3}{5}$$
$$6\tfrac{6}{7} \; is \; \tfrac{2}{7} \times 24$$
$$9 \; is \; 15 \times \tfrac{3}{5}$$
$$\left.\begin{array}{r}120 \\ 24\end{array}\right\} is \; 15 \times 24$$
$$\overline{375\tfrac{36}{35}} = 376\tfrac{1}{35}$$

$$2\tfrac{1}{3} \times 1\tfrac{1}{2} = \tfrac{7}{3} \times \tfrac{3}{2}$$
$$= \tfrac{7}{2}$$
$$= \; ?$$

EXERCISES

Find the following products:

1.	$24\tfrac{3}{5}$	**5.**	$14\tfrac{2}{5}$	**9.**	$32\tfrac{5}{6}$	**13.**	$21\tfrac{2}{5}$
	$15\tfrac{1}{4}$		$15\tfrac{5}{6}$		$\tfrac{7}{8}$		$15\tfrac{3}{8}$
2.	$18\tfrac{5}{6}$	**6.**	$16\tfrac{2}{3}$	**10.**	$48\tfrac{4}{7}$	**14.**	$27\tfrac{4}{5}$
	$3\tfrac{3}{9}$		9		$\tfrac{7}{16}$		$25\tfrac{5}{6}$
3.	$35\tfrac{3}{4}$	**7.**	$23\tfrac{8}{15}$	**11.**	$7\tfrac{3}{8}$	**15.**	$28\tfrac{1}{3}$
	$16\tfrac{2}{5}$		25		$16\tfrac{3}{5}$		$19\tfrac{1}{5}$
4.	$18\tfrac{2}{7}$	**8.**	$21\tfrac{2}{3}$	**12.**	$34\tfrac{7}{12}$	**16.**	$12\tfrac{5}{6}$
	$21\tfrac{5}{6}$		$\tfrac{4}{7}$		$24\tfrac{3}{5}$		$3\tfrac{4}{5}$

17. Find the area of a circle by multiplying its circumference, $2\pi R$, by $\tfrac{1}{2}R$.

18. Find the area of a rectangle by multiplying its length, $12r$, by its width, $\tfrac{3}{4}r$.

19. How much dirt will be taken out of a rectangular basement that is $42f$ long, $24f$ wide, and $3\tfrac{2}{3}f$ deep?

20. Express the cost of d dozen articles at \$ $\tfrac{3}{4}a$ each.

73. Multiplication with $\frac{1}{2}$.—If both fractions of mixed numbers are $\frac{1}{2}$, the integer of their product is the product of the integers plus $\frac{1}{2}$ the sum of the integers. To this is annexed $\frac{1}{4}$ if the integers are both even or both odd, and $\frac{3}{4}$ if one is even and the other odd.

$$\begin{array}{l} 35\frac{1}{2} \\ \underline{47\frac{1}{2}} \\ \quad\frac{1}{4}\ is\ \frac{1}{2}\times\frac{1}{2} \\ 41\quad is\ \frac{1}{2}(35+47) \\ \left.\begin{array}{l}245 \\ 140\end{array}\right\}is\ 47\times35 \\ \overline{1686\frac{1}{4}} \end{array}$$

$$\begin{array}{l} 26\frac{1}{2} \\ \underline{37\frac{1}{2}} \\ \quad\frac{1}{4}\ is\ \frac{1}{2}\times\frac{1}{2} \\ 31\frac{1}{2}\ is\ \frac{1}{2}(26+37) \\ \left.\begin{array}{l}182 \\ 78\end{array}\right\}is\ 37\times26 \\ \overline{993\frac{3}{4}} \end{array}$$

74. Multiplying by a Fraction Near a Unit.—To multiply by $29\frac{3}{4}$, multiply by 30 and subtract $\frac{1}{4}$ of the number. Why does this give the correct result? Carry out the operations to the right.

$$\begin{array}{l} 58 \\ \underline{29\frac{3}{4}} \\ \overline{1740}\ \ is\ 30\times56 \\ \quad 14\frac{1}{2}\ is\ \frac{1}{4}\times58 \\ \overline{1725\frac{1}{2}} \end{array}$$

EXERCISES

Carry out the following operations:

1. $24\frac{1}{2}\times32\frac{1}{2}$
2. $57\frac{1}{2}\times39\frac{1}{2}$
3. $67\frac{1}{2}\times23\frac{1}{2}$
4. $45\frac{1}{2}\times56\frac{1}{2}$
5. $29\frac{1}{2}\times32\frac{1}{2}$

6. $35\frac{1}{2}\times28\frac{1}{2}$
7. $29\frac{1}{2}\times36\frac{1}{2}$
8. $56\times39\frac{1}{2}$
9. $48\times19\frac{3}{4}$
10. $64\times\frac{7}{8}$

11. $72\times\frac{5}{6}$
12. $137\times29\frac{3}{4}$
13. $345\times\frac{7}{8}$
14. $637\times19\frac{3}{4}$
15. $756\times99\frac{3}{4}$

16. Find the cost of 2450 bu. wheat at \$ 1.49$\frac{3}{4}$ per bushel; of 1980 bu. at \$ 1.98$\frac{1}{2}$.

17. At 45 ¢ per hour what will be paid for 39$\frac{1}{2}$ hr. work? for 58$\frac{3}{4}$ hr.? for 37$\frac{1}{4}$ hr.? for 134$\frac{3}{4}$ hr.?

18. What is the cost of 167 doz. eggs at 39$\frac{3}{4}$ ¢ per dozen? *135 doz. at* 29$\frac{1}{2}$ ¢ per dozen? 119 doz. at 33$\frac{1}{3}$ ¢ per dozen?

75. Dividing a Fraction by an Integer.—To divide a fraction by an integer, divide the numerator—the numbering part of the fraction—by the integer. Thus, $\dfrac{6}{7} \div 3 = \dfrac{6 \div 3}{7} = \dfrac{2}{7}$. If the denominator had been multiplied by the divisor and the resulting fraction reduced to lowest terms, the same quotient would have been obtained. The latter process is used whenever the numerator cannot be divided by the integral divisor. In practice, merely multiply by a fraction having one as numerator and the integer as denominator. Thus,

$$\tfrac{6}{7} \div 4 = \tfrac{6}{7} \times \tfrac{1}{4} = ?$$

76. Dividing by a Fraction.—How many groups of 3 in each group can be made from 12 objects? How many groups of $\tfrac{1}{3}$ in each group can be made from 12 objects? Hence, to divide by a unit fraction merely multiply by the denominator. Thus,

$$\tfrac{3}{4} \div \tfrac{1}{6} = \tfrac{3}{4} \times 6 = ?$$

To divide by $\tfrac{7}{6}$, divide by $7 \times \tfrac{1}{6}$. That is, divide first by 7, which is the same as multiplying by $\tfrac{1}{7}$, and then divide by $\tfrac{1}{6}$, which is the same as multiplying by 6. The whole process is, then, the same as multiplying by $\tfrac{6}{7}$, or by the original divisor inverted. Thus,

$$\frac{35}{18} \div \frac{7}{6} = \frac{\overset{5}{\cancel{35}}}{\underset{3}{\cancel{18}}} \times \frac{\cancel{6}}{\cancel{7}} = \frac{5}{3} = ?$$

EXERCISES

Carry out the following operations:

1. $\tfrac{6}{7} \div 3$	5. $\tfrac{11}{8} \div \tfrac{33}{32}$	9. $16\tfrac{7}{8} \div \tfrac{5}{6}$
2. $\tfrac{10}{11} \div 5$	6. $\tfrac{35}{12} \div \tfrac{21}{16}$	10. $54\tfrac{5}{8} \div \tfrac{20}{27}$
3. $\tfrac{45}{26} \div 9$	7. $\tfrac{13}{32} \div \tfrac{35}{12}$	11. $15\tfrac{7}{8} \div \tfrac{4}{5}$
4. $\tfrac{15}{8} \div 20$	8. $\tfrac{23}{4} \div \tfrac{69}{8}$	12. $10\tfrac{3}{4} \div \tfrac{5}{6}$

77. Dividing by a Mixed Number.—To divide by a mixed number, reduce it to an improper fraction, invert, and multiply. Thus,

$$24\tfrac{3}{5} \div 2\tfrac{1}{3} = 24\tfrac{3}{5} \div \tfrac{7}{3} = 24\tfrac{3}{5} \times \tfrac{3}{7} = \,?$$

Note that it is not necessary to change the dividend to an improper fraction.

<div align="center">EXERCISES</div>

Carry out the following divisions:

1. $35 \div 3\tfrac{1}{2}$	**5.** $12\tfrac{2}{3} \div 2\tfrac{4}{5}$	**9.** $18\tfrac{3}{5} \div 3\tfrac{3}{5}$
2. $42 \div 2\tfrac{4}{5}$	**6.** $16\tfrac{1}{5} \div 1\tfrac{3}{5}$	**10.** $24\tfrac{1}{7} \div 2\tfrac{1}{3}$
3. $56 \div 2\tfrac{1}{3}$	**7.** $25\tfrac{3}{8} \div 2\tfrac{1}{4}$	**11.** $36\tfrac{2}{7} \div 5\tfrac{1}{2}$
4. $48 \div 3\tfrac{1}{5}$	**8.** $19\tfrac{1}{5} \div 1\tfrac{1}{4}$	**12.** $32\tfrac{3}{4} \div 2\tfrac{1}{3}$

13. A merchant owning $\tfrac{3}{5}$ of a stock of goods contemplates selling $\tfrac{2}{3}$ of his share. What part of the whole stock would he sell? What part of the whole stock would he still own?

14. What part of a yard is 6 in. ? 9 in. ? 15 in. ? 20 in. ?

15. Find the length of a rectangle having an area $35y^2$ and a width $3\tfrac{1}{2}y$; having an area $13\tfrac{3}{4}r^2$ and a width $5\tfrac{1}{2}r$; having an area $12\tfrac{2}{3}m^2$ and a width $2\tfrac{2}{3}m$.

16. Find the cost of $25\tfrac{3}{4}$ yd. of cloth at $\$ 2\tfrac{3}{5}$ per yard; of $34\tfrac{2}{3}$ yd. at $\$ 1\tfrac{3}{4}$ per yard.

17. Every pound of butter fat makes $1\tfrac{1}{6}$ lb. of butter. How many pounds of butter can be made from $12\tfrac{1}{2}$ lb. of butter fat? from 15 lb. butter fat? from $8\tfrac{1}{4}$ lb. butter fat?

18. A broker bought a stock of goods for $\tfrac{3}{5}$ of the invoice price and sold them for $\tfrac{3}{4}$ of the invoice price. What part of the invoice price did he gain?

78. Fractions a Form of Division.— $\frac{12}{4}$ means 12 of the $\frac{1}{4}$'s of a number. Since it takes 4 of these to make a whole one there are 3 whole ones in $\frac{12}{4}$. But that is also obtained from $12 \div 4$. Hence, $\frac{12}{4} = 12 \div 4$.

79. Solution of Problems.—Problem solutions may be greatly simplified by first merely indicating the various operations and then later carrying out these operations.

Find the cost of 54 cans of salmon at $ 3.20 per dozen.

The number of dozen is expressed by $\frac{54}{12}$, which is multiplied by the price per dozen. This gives the price as

$$\frac{54}{12} \times \$ 3.20.$$

Simplify this by cancellation.

This method, or procedure, has two big advantages:

1. *First, all the attention can be directed toward the thought process of the problem.*

2. *Later, all the attention can be directed toward the numerical part of the solution. In the latter part, time may also be saved by taking advantage of the simplifications arising through cancellation.*

<div align="center">

EXERCISES

</div>

Indicate all operations before carrying out any of the numerical work.

1. Find the cost of 32 cans of pineapple at $ 4.80 per dozen.

2. What is the cost of 1840 lb. of wheat at $ 1.80 per bushel ?

3. If there are 48 lb. of apples to the bushel, what will be the cost of 340 lb. apples at $ 1.60 per bushel ?

4. Last year a farmer raised 230 bu. wheat from 14 acres. How many bushels can he expect this year from 80 acres ? from 50 acres ? from 160 acres ?

IV

APPROXIMATIONS

80. Estimates.—"About how much will this be ? " or "About how much will that cost ? " are questions that arise more times than the exact ones of "**How much** will this be ?" or "**How much** will that cost ?" As an illustration, may be cited the question of finding **about** how much a railroad ticket will cost at 3 ¢ per mile for 387 mi. As it lacks about 10 mi. of being 400 mi., the price of the ticket will lack a little more than 30 ¢ of being the cost of a ticket for 400 mi. Hence the cost will be nearly $ 12 less 30 ¢, or about $ 11.70.

81. Estimations as Checks.—Estimates serve as excellent checks on computations. In estimates the numerical work is simpler than the exact computations and for that reason the estimation will generally give a result fairly close to the true value. If the computed value differs widely from the estimated value, it is quite certain that the computed value is incorrect. Always use this method of checking your solution of problems in the future.

EXERCISES

Estimate the following values:

1. 35×42
2. 28×62
3. 75×24
4. 26×64
5. 47×52
6. 37×48

7. 105×97
8. 74×48
9. $23\frac{1}{2} \times 47\frac{1}{2}$
10. $59 \times 51\frac{3}{4}$
11. $35\frac{7}{8} \times 33\frac{3}{5}$
12. $16\frac{7}{8} \times 24\frac{2}{5}$

13. $68\frac{3}{4} \times 52\frac{1}{4}$
14. $77\frac{5}{6} \times 82\frac{1}{5}$
15. $67\frac{2}{3} \times 54\frac{3}{4}$
16. $17\frac{2}{3} \times 23\frac{4}{5}$
17. $37\frac{1}{5} \div 8$
18. $59\frac{4}{5} \div 5\frac{2}{4}$

82. Short Cuts in Estimates.—Short cuts should be used frequently in estimates, as elsewhere. For instance, 26 × 64 is nearly 25 × 64, which is ¼ of 6400. How?

EXERCISES

1. If use was not made of short cuts in making the estimates called for in the exercises at the bottom of the last page, turn back to them and apply short cuts to as many as possible.

2. Estimate the results called for on pages 10 and 12.

3. Estimate the results called for on page 40.

4. Estimate the results called for on page 42.

83. Meaningless Numbers.—In finding the price of several bushels of grain at 87⅜ ¢ per bushel, the ⅜ ¢ has a meaning, but in the price of only **one** bushel the ⅜ ¢ has no meaning. Why?

In United States money it is customary to count ½ or a larger fraction of a cent as one cent and to drop all smaller fractions of a cent. Thus, $ 4.536 is $ 4.54 while $ 4.532 is $ 4.53.

This plan is used in general. Thus, to the nearest **whole foot** 64 ft. 7 in. is called 65 ft., while 64 ft. 3 in. is called 64 ft. In computations where only two decimal places can be used 34.5873 would be called 34.59 while 34.5848 would be called 34.58.

EXERCISES

1. State how much will pay the following bills: $ 12.56½; $ 3.463; $ 72.684; $ 120.476; $ 132.732.

2. State the following lengths to the nearest foot: 3 ft. 4 in.; 6 ft. 8 in.; 38 in.; 34 in.; 27 in.; 7 ft. 4.6 in.

3. State the following ages to the nearest year: 18 yr. 5 mo.; 34 yr. 5½ mo.; 47 yr. 6 mo.; 28 yr. 6¾ mo.; 19 yr. 190 da.; 41 yr. 150 da.

4. State the following to two decimal places: 45.063; 27.186; 108.135; 304.045; 31.008; 19.093; 71.058.

5. State the following populations to the nearest thousand: 15,200; 26,750; 135,450; 6,530.

84. Abbreviated Multiplication.—Suppose that the product of 69.37 × 28.45 is desired to two decimal places. Instead of carrying out all of the work and dropping the last digits, the work can be abbreviated as is here shown.

```
    28.45
    69.37
   ───────
   256.05  (1)
  1707.00  (2)
     8.54  (3)
     1.99  (4)
  ───────
  1973.58  (5)
```

1. Always multiply by unit's digit first to locate the decimal point.

2. 60 × 28.45 in the usual manner.

3. In .3 × .05 = .015 the 5 is not written but 2 in the place of 1 carried to the multiplication of .3 × .4. Then complete as usual. Draw a line through the 5 to show that it is not used in multiplying by .07.

4. .07 × .05 is not considered. In .07 × .4 = .028 the 8 is not written, but 3 in the place of 2 carried to .07 × 8. Then as usual.

EXERCISES

Carry out the following to two decimal places:

1.	23.25 × 34.16	**7.**	45.27 × 30.15
2.	16.07 × 52.13	**8.**	26.15 × 46.08
3.	47.53 × 79.34	**9.**	24.15 × 57.04
4.	24.16 × 74.25	**10.**	42.45 × 63.54
5.	34.26 × 53.17	**11.**	24.05 × 67.16
6.	32.23 × 67.18	**12.**	54.05 × 74.60

85. Abbreviated Divisions.—In the place of annexing zeros to the dividend in carrying out a division to decimals, digits are cut off from the divisor.

$$\begin{array}{r} 2.65 \\ \hline 2563)6790 \\ 5126 \quad (1) \\ \hline 1664 \quad (2) \\ 1538 \quad (3) \\ \hline 126 \quad (4) \\ 128 \quad (5) \\ \hline \end{array}$$

1. 2 × 2563, in the usual manner.

2. Cut off 3 in divisor. 1664 ÷ 256 gives a quotient of 6.

3. In 6 × 3 = 18 the 8 is not written but 2 in the place of 1 carried to 6 × 6. Then complete multiplication as usual.

4. Cut off 6 in divisor. 126 ÷ 25 gives 5 as the nearest quotient.

5. In 5 × 6 = 30 the 0 is not written and the 3 merely carried to 5 × 25.

EXERCISES

Carry out the following to two decimal places:

1. 7248 ÷ 3112
2. 8415 ÷ 2415
3. 4007 ÷ 1034
4. 7604 ÷ 718
5. 5607 ÷ 2814
6. 8408 ÷ 1054
7. 8456 ÷ 4012
8. 3006 ÷ 213
9. 2034 ÷ 435
10. 5041 ÷ 3047
11. 870.5 ÷ 23.45
12. 56.34 ÷ 32.04

13. Find the cost of 845 lb. sugar at $ 7.65 per cwt.

14. Find the cost of 735 lb. beef at $ 27.45 per cwt.

15. A farmer is offered $ 9500 for 324 acres of land. How much is this per acre ?

16. What is the area to the nearest square foot of a rug that is 14.5 ft. by 17.3 ft. ?

17. A coal and wood dealer bought some growing timber from which he cut 423 cords of wood. If the total expense was $ 1756, what did he pay per cord for the wood?

V

PERCENTAGE

86. Meaning of Percentage.—We have already learned that, several hundred years before the invention of decimals, merchants found it convenient to count by the hundred. In this system $\frac{2}{5}$, or .4, becomes $\frac{40}{100}$. A per cent is therefore merely the numerator of a fraction having 100 as its denominator. Similarly, $\frac{3}{25}$, or .12, which reduces to $\frac{12}{100}$, is called 12 **per cent** and is written 12 %. Again,

$$\frac{1}{8} = \frac{12\frac{1}{2}}{100} = 12\frac{1}{2}\ \%.$$

As percentage is a part of common business language, it should be understood thoroughly by all.

87. Changing a Common Fraction or a Decimal to a Per Cent.—First change the common fraction or decimal to a common fraction with a denominator 100, then write the numerator alone with the per cent sign after it. Thus,

$$\frac{3}{5} = \frac{60}{100} = 60\ \%; \qquad \frac{1}{3} = \frac{33\frac{1}{3}}{100} = 33\frac{1}{3}\ \%;$$

$$.45 = \frac{45}{100} = 45\ \%; \quad 2.05 = \frac{205}{100} = 205\ \%.$$

88. Changing a Per Cent to a Common Fraction or to a Decimal.—Write the per cent as the numerator of a common fraction with 100 as the denominator. Then reduce this to its lowest terms or to a decimal. Thus,

$$15\ \% = \frac{15}{100} = \frac{3}{20}; \qquad 12\frac{1}{2}\ \% = \frac{12\frac{1}{2}}{100} = \frac{1}{8};$$

$$60.3\ \% = \frac{60.3}{100} = .603; \quad 135\ \% = \frac{135}{100} = 1.35.$$

EXERCISES

Fill in the blank spaces in the columns given below. Carry out mentally as many as possible.

1. $\frac{4}{5} =$. $? = ? \%$
2. $\frac{7}{6} =$. $? = ? \%$
3. $? = .45 = ? \%$
4. $? = .08\frac{1}{2} = ? \%$
5. $? = .68 = ? \%$
6. $? =$. $? = 25 \%$
7. $2\frac{1}{2} =$. $? = ? \%$
8. $? = 3.5 = ? \%$
9. $? = 1.005 = ? \%$
10. $? =$. $? = 15\frac{1}{2} \%$
11. $? =$. $? = 500 \%$
12. $? =$. $? = 9\frac{1}{5} \%$
13. $4\frac{1}{3} =$. $? = ? \%$
14. $3\frac{1}{4} =$. $? = ? \%$
15. $? = 1.05 = ? \%$
16. $\frac{9}{12} =$. $? = ? \%$
17. $? =$. $? = 7\frac{1}{2} \%$
18. $? =$. $? = 10.6 \%$
19. $? =$. $? = 125\frac{1}{2} \%$
20. $? = 1.05\frac{1}{2} = ? \%$

21. $\frac{3}{4} =$. $? = ? \%$
22. $\frac{3}{8} =$. $? = ? \%$
23. $\frac{8}{15} =$. $? = ? \%$
24. $? =$. $? = 48 \%$
25. $? = .125 = ? \%$
26. $? =$. $? = 35 \%$
27. $? = .08\frac{1}{4} = ? \%$
28. $? = .005 = ? \%$
29. $? =$. $? = 12\frac{1}{2} \%$
30. $? =$. $? = 135 \%$
31. $? =$. $? = 6.3 \%$
32. $? =$. $? = 108 \%$
33. $\frac{4}{7} =$. $? = ? \%$
34. $? = 4.02 = ? \%$
35. $\frac{1}{3} =$. $? = ? \%$
36. $? =$. $? = 87\frac{1}{2} \%$
37. $? = .205 = ? \%$
38. $? = .03\frac{1}{4} = ? \%$
39. $1\frac{2}{3} =$. $? = ? \%$
40. $? =$. $? = 8\frac{1}{3} \%$

41. A nickel is what per cent of a dollar? of a half-dollar? of a quarter? of a dime? of 20 ¢? of 40 ¢? of 30 ¢?

42. What per cent of a yard is 9 in.? 1 ft.? 18 in.?

43. What per cent of a foot is 3 in.? 6 in.? 8 in.? 2 in.? 15 in.?

89.   Finding a Per Cent of a Number.—20 % of 450 is the same as $\frac{1}{5}$ or .2 of 450.   Why ?   In finding a per cent of a number the per cent must first be reduced either to a common fraction or to a decimal.   It is simpler to change the per cent to a decimal except when the equivalent fraction is a simple aliquot part of 100, as in the list here given.

$6\frac{1}{4}$ %	$16\frac{2}{3}$ %	$33\frac{1}{3}$ %	$66\frac{2}{3}$ %
10 %	20 %	$37\frac{1}{2}$ %	75 %
$12\frac{1}{2}$ %	25 %	50 %	$87\frac{1}{2}$ %

The per cent is also the **rate per cent.**

Find 45.2 % of 352.   First change 45.2 % to .452 and then find .452 of 352.   Hence, what is 45.2 % of 352 ?

Find $16\frac{2}{3}$ % of 624.   First notice that $16\frac{2}{3}$ % is the common fraction $\frac{1}{6}$.   Then find $\frac{1}{6}$ of 624.   What is this ?   Hence, what is $16\frac{2}{3}$ % of 624 ?

Finding n, which is R per cent of N, is stated by the equation,

$$n = R \% \times N. \qquad \text{How ?}$$

The number n is sometimes called **percentage.**

EXERCISES

1.   Change each per cent in the list above to a common fraction; to a decimal.   When is it better to use the common fraction than the decimal form in solving problems containing a per cent ?

Use pencil and paper as little as possible.

2. 25 % of 412	**8.** 30.6 % of 1605
3. 56 % of 519	**9.** $33\frac{1}{3}$ % of 1428
4. $12\frac{1}{2}$ % of 1648	**10.** $66\frac{2}{3}$ % of 516
5. 45.6 % of 3405	**11.** 23.5 % of 1250
6. 35 % of 804	**12.** $26\frac{1}{2}$ % of 7042
7. 35.4 % of 9735	**13.** $34\frac{1}{5}$ % of 1678

14. 125 % of 3672   **17.** 133⅓ % of 456
15. 115 % of 4056   **18.** 200 % of 34059
16. 205 % of 3405   **19.** 108½ % of 3408

20. In 1918 the government increased freight rates 25 %. What would be the increase in a freight bill which before had been $35.36 ? $106.32 ? $85.46 ? $124.08 ?

21. The new freight, F, is what per cent of the old freight, f ? State this as an equation.

22. From the equation in Ex. 21 find f in terms of F. Hence, the old freight is what per cent of the new freight ?

23. A manual on home architecture states that in a well-proportioned living-room the width should be 75 % of the length. State W in terms of L by an equation. Find L in terms of W.

24. What should be the width of a living-room that is 16 ft. long ? 18 ft. long ? 22 ft. long ? 25 ft. long ?

25. In a well-lighted room the window-space is at least 18 % of the floor-space. How large should the window-space be in a room 9 by 10 ft. ? 10 by 12 ft. ? 15 by 21 ft. ?

26. Apply the test in Ex. 25 to your schoolroom.

27. About 23 % of the total population of the United States is of school age. Find the number of school-children in the following states with the populations stated:

Massachusetts.................3,366,000
Indiana 2,700,000
Minnesota....................2,075,000
Virginia.....................2,062,000
Washington...................1,142,000

28. The milk from a certain cow contains 3.5 % butter fat. How many pounds of butter fat in 416 lb. milk ?

29. Cork weighs 24 % as much as an equal volume of water. If a cubic foot of water weighs 62.5 lb., what will be the weight of a cubic foot of cork ?

30. The government estimates that $7\frac{1}{2}$ % of the money spent for food is wasted. At this rate how much is wasted per year in a home where the monthly expenditure for food is $ 26 ? $ 25 ? $ 35 ? $ 75 ?

31. In a certain junior high school 96 % of the pupils invested in Thrift Stamps and Baby Bonds. What per cent of the school did not invest in Thrift Stamps or Baby Bonds ? At this rate how many of the estimated 23,000,000 school-children in this country own Thrift Stamps or Baby Bonds ?

32. About 65 % of the current necessary tò run an electric toaster is wasted. What per cent is used ? State by an equation that c units of current are used of C units paid for. How much current of 30 ¢ paid for is used ?

33. Solve the above equation for C. How much must be paid for the current tò get 40 ¢ of usable current ?

34. Solve the equation on page 50 for N.

90. **Finding a Number that is a Certain Per Cent of Another Number.**—We have just been finding the product of two numbers, one of which was a per cent expressed as a common fraction or as a decimal. We have here the reverse problem: namely, to find the number which, multiplied by the per cent, produces the given product number. This is at once seen from the equation,

$$N \times R\ \% = n. \qquad (1)$$

Hence,
$$N = n \div R\ \%. \qquad (2)$$

Twenty-five per cent of what number equals 315 ?

Here $R\ \% = .25$, $n = 315$, and the number, N, is found to be

$$N = 315 \div .25 \qquad \text{How ?}$$
$$= ? \qquad \text{How ?}$$

EXERCISES

1. Find the number of which 30 % is 216.

2. Find the number of which 32 % is 360.

3. Find the number of which 12½ % is 816.

4. Of what number is 45.4 % equal to 908 ?

5. According to Ex. 23, page 51, what is the appropriate length of a room 12 ft. wide ? 16 ft. wide ? 18 ft. wide ?

6. Milk is about 4 % fat. How many pounds of milk will it take to give 12 lb. fat ? 10 lb. fat ? 50 lb. fat ?

7. How much milk which contains 3.7 % butter fat will be needed for 50 lb. butter fat ? 30 lb. butter fat ?

8. A cow gives milk that is 4.1 % butter fat. How many pounds of this milk are necessary to produce 100 lb. butter fat ? 60 lb. butter fat ?

9. Butter contains 85 % fat. How many pounds of butter will it take to give 15 lb. fat ? 50 lb. fat ?

10. Canned salmon contains 12.1 % fat. How much fat is there in a 12-oz. can of salmon ? in an 8-oz. can ?

11. How many pounds of salmon are necessary to give 6 oz. fat ? 5 lb. fat ? 25 lb. fat ?

12. Potatoes contain 75.5 % water. State by an equation the number of pounds of water, w, in p pounds of potatoes. How many pounds water in a bushel, 60 lb., of potatoes ? How many pounds potatoes give 80 lb. water ?

13. John has attained a spelling average of 96 % during the year in school. What does this mean ? How many words would he miss out of every 100 at this rate ?

14. At the above rate how many words would John miss out of 250 tried ? 375 tried ?

15. In a spelling contest John missed 12 words. At the rate in Ex. 13, how many words was he asked to spell ?

16. Solve the equation on page 50 for R %.

91. Finding What Per Cent One Number is of Another Number.—First find what decimal part one number is of another given number. This is really finding by what decimal one number must be multiplied to produce the other number. The decimal thus found is then changed to a per cent. Thus, 24 is what per cent of 96 ?

Solving for $R \%$ in $\qquad R \% \times N = n,$ $\qquad$ (1)
gives $\qquad\qquad\qquad R \% = n \div N.$ $\qquad$ (2)
Hence, for this problem, $R \% = 24 \div 96 = .25 = 25 \%.$

The width is what per cent of the length in a room 14 ft. by 16 ft. ?
From (2) $\qquad R \% = \frac{14}{16} = .875 = 87\frac{1}{2} \%.$

EXERCISES

Find what per cent

1.	18 is of 72	**7.**	24 is of 246	**13.**	60 is of 15
2.	15 is of 45	**8.**	45 is of 265	**14.**	250 is of 50
3.	24 is of 144	**9.**	34 is of 308	**15.**	45 is of 246
4.	36 is of 180	**10.**	18 is of 246	**16.**	90 is of 340
5.	16 is of 48	**11.**	35 is of 210	**17.**	85 is of 974
6.	25 is of 225	**12.**	36 is of 324	**18.**	925 is of 245

19. John spelled 15 out of 18 words correctly. What is his per cent; that is, what per cent of the total number of words did he spell correctly ?

20. If Mary solved 18 out of 20 problems in her mathematics lesson, what per cent did she solve ?

21. Out of 319 problems assigned to Henry during one month in his mathematics class he solved 297. What per cent of the problems assigned did Henry solve correctly ?

22. At the rate found in Ex. 21, how many problems could Henry be expected to work in an assignment of 14 ?

23. Jane solved 287 of the 296 problems assigned to her one month. What per cent of the problems assigned did Jane solve? Compare this with Henry's record in Ex. 21.

24. William bought a paper route having 98 customers. The first month he added 14 new customers. What per cent of the original customers did he add the first month?

25. How many customers has William after adding n new ones to his old ones, O? These are what part of the new customers? These are what part of the old customers? The old are what part of the new customers?

26. A city in the United States was asked to subscribe $25,000 to the Red Cross. It actually subscribed $48,500. The subscription was what per cent of its quota? The subscription was what per cent above the quota?

27. In a certain junior high school of 251 pupils it was found that 236 of the pupils owned Thrift Stamps or Baby Bonds. What per cent of the pupils owned Thrift Stamps or Baby Bonds?

28. In another junior high school of 567 pupils it was found that 523 pupils owned Thrift Stamps or Baby Bonds. What per cent of the pupils owned Thrift Stamps or Baby Bonds? Which school had the better per cent?

29. In a junior high school of 437 pupils 49 were absent one-half day or more during a month of 20 da. What per cent of the total number of pupils had been present during that month? What per cent of the pupils had been absent during that month?

The population of a certain city was 34,000 in 1900 and 41,500 in 1910.

30. The population in 1910 was what per cent of the population in 1900?

31. The population in 1900 was what per cent of the population in 1910?

92. Per Cents of Increase or Decrease.—If a number is increased a certain per cent, r, of itself, this amount is added to the number.   This is expressed by the equation,

$$N = n + nr. \qquad (1)$$

Hence, $\qquad \qquad N = (1 + r)n. \qquad (2)$

Explain the meaning of equation (1) and show how equation (2) is derived from (1). If a number is decreased a certain per cent, r, of itself, the equation will be

$$N = n - nr. \qquad (3)$$

Hence, $\qquad \qquad N = (1 - r)n. \qquad (4)$

Explain equation (3) fully and show how equation (4) is derived from it.

What will be the result of increasing 36 by 13 % of itself ?

$$N = 36(1 + .13) = 36 \times 1.13 = ?$$

In raising chickens, Henry found that he usually lost about 15 % of those hatched.   Out of 86 chickens hatched how many could he count on maturing ?

$$N = 86(1 - .15) = 86 \times .85 = ?$$

EXERCISES

1.   What will be the result of increasing 36 by 25 % of itself ? by 15 % of itself ?

2.   What will be the result of decreasing 48 by 25 % of itself ? by 32 % of itself ?

3.   How much is 45 increased by 24 % of itself ? by 50 % of itself ? by 40 % of itself ?

4.   How much is 84 decreased by 25 % of itself ? by 45 % of itself ? by 35 % of itself ?

5.   How much is 296 increased by 20 % of itself ? decreased by 15 % of itself ?

6. Last year Henry raised chickens and lost 35 % of all that hatched. How many did he mature out of 97 hatched ?

7. This year Henry will be more careful with his chickens and expects to lose not more than 15 % of those hatched. What number will he then raise out of 97 hatched ?

8. What per cent of the total number hatched does he expect to raise this year more than he did last year ? How many chickens would that be of the number hatched last year ?

9. Last year Mary earned $ 13.45 on her garden. This year she has increased her garden plot 50 %. By what per cent could she expect to increase her income from her garden this year ? How much money does she expect to derive from her garden this year ?

10. Jack bought a paper route having 56 customers. By giving the best of service and looking for new customers, he expects to increase this number 25 %. How many customers would he then have ?

11. The value of the production of a factory during the year preceding the war was $ 135,400. The next year the production was decreased 65 %. What was the value of the production that year ?

12. Last year Harry saved $ 45.75 that he invested in Baby Bonds and Thrift Stamps. Next year he expects to earn more and to be more careful about his expenditures; then he expects to invest 25 % more. How much will he invest next year ? What will his total savings be ?

13. A certain county in one of the western states was asked to subscribe $ 25,000 to the Red Cross. They decided to oversubscribe this by at least 75 %. What would that make their subscription ?

14. Solve equation (1), page 56, for n.

15. What number increased by 25 % of itself will give 48 ? 36 ? 324 ? 215 ?

16. What number increased by 32 % of itself will give 96 ? 406 ? 346 ?

17. The expenses for running a store a year were $ 11,845, which was estimated to be 25 % above the previous year. What were the expenses the previous year ?

18. The diagonal through a square is 41 % longer than one of its equal sides. In passing through a square Henry counted 346 steps. How many steps of the same length would he have taken in walking one side of the square ?

19. Solve equation (3), page 56, for n.

20. What number decreased by $33\frac{1}{3}$ % of itself will give 56 ? 128 ? 345 ?

21. What number decreased by 18 % of itself will give 164 ? 410 ? 916 ?

22. Last year Mr. Worly did not test his seed corn, and got 34 bu. per acre. When he tested his seed this spring he found that 26 % was not seed. If the same per cent of the corn planted last year was not seed, what would have been his yield if all the corn had been seed ?

23. In raising chickens William has found that an average of 18 % of the eggs set did not develop into mature chickens. What per cent of the eggs matured into grown chickens ? At this rate how many eggs would he need to set in order to raise 50 chickens ? 75 chickens ?

24. A church society deciding to ask for subscriptions estimated that 15 % of the subscriptions would not be paid. How large a subscription would they need in order to raise $ 2,500 ?

25. Solve equation (1), on page 56, for r.

26. Find the per cent of increase of a number changed from 20 to 30; 36 to 48; 25 to 35.

27. John bought a paper route having 46 customers. He increased the number of customers to 84. What per cent of increase was this?

28. At the beginning of a year Sara weighed 86 lb. and at the end of the year she weighed 91 lb. By what per cent had she increased her weight?

29. Last year Cora saved $ 32.50 of the money she had earned working during vacation and after school. This year she saved $ 54.25. What was her per cent of increase in savings this year over last year?

30. Solve equation (3), page 56, for r.

31. Find the rate of decrease of a number changing from 312 to 220; 150 to 125; 120 to 80.

32. Mr. Jackson burned $ 95 worth of coal last year and only $ 72 this year. What was the per cent of decrease in the cost of coal he used this year over last year?

33. By installing smoke-consumers a factory decreased its coal bill from $ 2450 one year to $ 1925 the following year. What was the per cent of decrease?

34. A large city school one month decreased its number of tardinesses to 89 from 137 for the previous month. What was the per cent of decrease?

35. The population of a certain city increased from 25,340 in 1900 to 27,815 in 1910. Find the per cent of increase.

36. The State of Iowa had a corn crop of 288,858,000 bu. in 1915 and of 377,235,000 bu. in 1916. What was the per cent of increase or decrease?

VI

INTEREST AND INVESTMENTS

93. Interest.—Interest is generally defined as, "Money paid for the use of money." **Interest is** then a **form** of **rent** paid for the use of money, just as rent is paid for the use of a house or an automobile.

There is yet another way to look at interest. Most of the money loaned at interest has been accumulated by some one who saved a part of his money in the place of spending it for any of the many things that he wanted. Looking at interest in this way it is **payment for the sacrifice of some one.** These sacrifices may not have been made by the one receiving and enjoying the interest, but the sacrifices **have** generally been necessary for the accumulation of any **money to loan** at interest.

94. History of Interest.—The payment of interest dates back to the ancient Hebrews, Greeks, and Romans. It originated in requiring those who did not settle their debts promptly to pay something extra. Like many other good customs, the payment of interest was abused and for a long time the collection of interest was looked down upon. In England it was forbidden by law from the time of King Alfred until the time of Henry.VIII. Many states of the Union have fixed maximum rates of interest—see page 235— that may be collected. A higher rate than that permitted by law is called **usury.** What would be usury in your state? What is the penalty for usury in your state?

95. Computation of Interest.—**Interest** is always a certain per cent, called **rate,** per year of the money loaned, which is called **principal.** The sum paid by the borrower at the end of the year, that is, principal and interest together, is called **amount.** If T expresses the time in years, R the rate as a decimal, and P the principal in dollars, then I in dollars becòmes,

$$I = PRT. \qquad (1)$$

Also $\qquad A = P + I \qquad (2)$

$$= P + PRT \qquad (3)$$

$$= P(1 + RT). \qquad (4) \quad \text{How ?}$$

The interest on \$ 350 at 7 % for 2 yr. will be

$$I = \$ 350 \times .07 \times 2 \quad \text{[Carry out mentally.]}$$

$$= \$ 49.$$

From (2) $\qquad A = \$ 350 + 49$

$$= \$ 399.$$

Note that the parentheses in (4) contain the amount of \$ 1 for the given rate and given time. This is then multiplied by the principal to give the total amount. Thus,

$$A = \$ 350(1 + .07 \times 2)$$

$$= \$ 350(1.14)$$

$$= \$ 399.$$

EXERCISES

Use pencil and paper as little as possible.

1. Find the interest on \$ 600 for 1 yr. at 8 %; at 7 %.

2. Find the interest on \$ 250 for 2 yr. at 5 %; at 3 %.

3. What will be the amount of \$ 1 for 1 yr. at 4 % ? at 8 % ?

4. What will be the interest and the amount for 1 yr. of \$ 200 at 6 % ? of \$ 450 at 4 % ? of \$ 6000 at 7 % ?

Find the interest and the amount for Exs. 5–13:

5. $ 750 for 2 yr. at 5 %.

6. $ 1250 for 3 yr. at 8 %.

7. $ 2510 for 1 yr. at $5\frac{1}{2}$ %.

8. $ 3240 for 2 yr. at $6\frac{1}{2}$ %.

9. $ 1600 for $1\frac{1}{2}$ yr. at 7 %.

10. $ 2700 for $1\frac{1}{3}$ yr. at 6 %.

11. $ 3245 for $1\frac{1}{4}$ yr. at $7\frac{1}{2}$ %.

12. $ 346.50 for $\frac{3}{4}$ yr. at 8 %.

13. $ 256.50 for $\frac{2}{3}$ yr. at 7 %.

14. Harriet owns $ 250, which she has invested in government bonds that pay her $4\frac{1}{4}$ % interest. What yearly interest does she receive? If the interest is paid semi-annually, how much is each payment?

15. Harry saved $ 45 from the money he earned during vacation and loaned it one year at $5\frac{1}{2}$ %. How much did the $ 45 earn for him during the year?

16. Mary saved $ 65 of the money she earned and had given to her during the year. How much interest will it earn her per year at 7 %? at 5 %? at $6\frac{1}{2}$ %?

17. William saved $ 85 of the money he earned during the year carrying papers. How much interest will it earn for him if loaned for a year at 6 %?

18. Jane has $ 124 in the savings-bank. How much will it amount to at the end of the year if the bank pays her 4 % interest? 3 % interest? $3\frac{1}{2}$ % interest?

In computing interest, months are first reduced to a fractional part of a year.

19. What fractional part of a year is 6 mo.? 4 mo.? 3 mo.? 2 mo.? 5 mo.? 9 mo.? 7 mo.?

Find the interest and the amount for Exs. 20–24:

20. $ 650 for 6 mo. at 4 %.

21. $ 1250 for 4 mo. at 6 %.

22. $ 645 for 5 mo. at 8 %.

23. $ 3450 for 1 yr. 4 mo. at 6 %.

24. $ 235 for 2 yr. 3 mo. at 7 %.

25. Solve equations (1) and (3), page 61, for T.

26. In how many years will $ 600 at 7 % give $ 105 interest ?

27. In how many years will $ 450 at 5 % give $ 25 interest ?

28. How long must $ 850 be placed at 5 % interest to amount to $ 935 ?

29. How long must $ 1250 be placed at 6 % interest to amount to $ 1427.50 ?

30. Henry has earned $ 50, which he loaned at 6 % interest. In how many years will the interest be $ 15 ? $ 25 ?

31. In how many years will Henry's money double itself ?

32. William has saved $ 65, which he loaned at $6\frac{1}{2}$ %. How long will it take this money to earn him $ 5 ? $ 12 ?

33. In what time will William's money amount to $ 100 ?

34. Show that for 1 da. the interest is $PR \frac{1}{360}$. State the formula for the interest for 1 mo.

35. Solve equations (1) and (4), on page 61, for P.

36. How much money loaned at 5 % for 2 yr. will give $ 45 interest ? $ 135 interest ? $ 205 interest ?

37. How much money loaned for 1 yr. at 6 % will amount to $ 2800 ? to $ 3000 ? to $ 650 ? to $ 1250 ?

38. How much money loaned for $1\frac{1}{2}$ yr. at 4 % will amount to $ 3180 ? to $ 477 ? to $ 662.50 ?

39. Solve equations (1) and (3), page 61, for R.

96. Rate of Interest.—In business it is often necessary to know what rate of interest is earned by money invested in some enterprise. Thus, a house valued at $ 3500 is rented so as to bring in an income of $ 216 after all expenses have been deducted, called **net income.** If the $ 216 is considered as the interest for 1 yr., what rate would the $ 3500 then bring? First solve equations (1) and (3), on page 61, for R. Now substitute the above values for P, I, and T and obtain the rate, R, from

$$R = \frac{I}{PT}.$$

EXERCISES

1. If $ 800 loaned for 1 yr. brings $ 48 interest, what is the rate?

2. At what rate has $ 650 been loaned if it brings in $ 42.50 interest per year?

3. During vacation John earned $ 45, which he loaned to his uncle one year for $ 3. At what rate of interest did John loan his money?

4. Henry owns a paper route for which he paid $ 125. After paying all his expenses and making an allowance for the time he works with his papers, he has $ 23.50 left at the end of the year. If he considers the $ 23.50 as interest on the $ 125 invested, what is the rate?

5. Mr. Jenkins owns a house that he values at $ 5500, which rents for $ 30 per month. His taxes are $ 55, insurance $ 12, and repairs $ 55 per year. What is his net income per year? This gives what rate of interest on the $ 5500 invested?

6. When money can be loaned at 6 % interest, can a man afford to rent his farm valued at $ 125 per acre so that it will net him $ 6.50 per acre yearly?

7. In a city where responsible business men could borrow money at 6 % interest, a loan agent loaned $ 1000 to each of seven firms at 10 % interest. Before the end of the year one firm failed and was able to pay only 60 % of the principal borrowed and no interest. What were the loan agent's complete returns? What rate of interest was this on the $ 7000 loaned? How does it compare with 6 % paid by responsible men?

8. Later we shall study carefully the reasons for different rates of interest. What reasons can you think of now?

97. Time Reckoned at a Bank.—Banks usually loan money for 30 da., 60 da., and 90 da. They reckon the exact number of days and not from one date to the same date in the second, third, or fourth month. Thus, 30 da. from July 5 would be to Aug. 4; 30 da. from Feb. 11 on ordinary years would be to Mar. 13; 60 da. from Apr. 9 would be to June 8; and so on. Explain each of the above statements. Banks reckon a month as 30 da.; hence, a year as 360 da.

EXERCISES

Give the exact date at which the following notes will be due:

1. Dated June 9 for 30 da.   5. Dated Mar. 3 for 2 mo.
2. Dated Oct. 7 for 60 da.   6. Dated Feb. 7 for 60 da.
3. Dated July 5 for 60 da.   7. Dated Nov. 4 for 90 da.
4. Dated Sept. 4 for 90 da.   8. Dated Oct. 7 for 6 mo.

9. On what date will a note be due that is dated to-day if it is to run for 60 da. ? for 90 da. ? for 30 da. ?

10. What is the exact number of days to your next birthday?

11. A 60-da. note for $ 560 is dated June 7, 1918. When is it due? What will it amount to at 7 %?

12. A 90-da. note for $ 725 is dated Mar. 14, 1918. When is it due? What must the man owing the note pay at that time at 6 % interest?

13. What will be the amount of $ 1250 placed at 7 % interest Aug. 12, 1919, if paid Oct. 11, 1919?

14. On May 29, 1918, Mr. Jackson loaned $ 1825 at 5½ % interest for 90 da. When and how much will he be paid?

15. Mr. Arnold borrowed $ 850 from the bank at 8 % interest on Feb. 18, 1900, for 60 da. When and how much must he pay the bank when the loan is due?

16. Mr. Wilson borrowed $ 1200 from his bank at 8 % on Sept. 7 for 90 da. If interest and principal are to be paid at the same time, when and how much will he pay the bank?

17. Write an equation stating the interest on $ d at r % for any number of 30-da. periods. Also write the corresponding equation for the amount. Solve each for r %.

18. When will $ 1425 borrowed to-day at 6½ % for 90 da. be due and what will be the interest?

19. When will $ 2450 borrowed to-day at 8 % for 60 da. be due and what will be the interest?

98. Bankers' Method.—The following is one of the simplest short cut methods of computing interest. It is based upon the fact that at 6 % the interest for 2 mo. will be 1 % of the principal. Why is this so? What is the interest at 6 % for 2 mo. on $ 4560? on $ 2350? on $ 1245?

Study the following carefully:

Find the interest at 6 % for 1 yr. 3 mo. 10 da. on $ 2450.

Interest 1 yr.................................$ 147.00
"   ₁ 2 mo.................................. 24.50
"   1 mo. ($\frac{1}{2}$ of 2 mo.)........ .... 12.25
"   10 da. ($\frac{1}{3}$ of 1 mo.)........................   4.08
                                            ‾‾‾‾‾‾‾‾
                                            $ 187.83

If the rate had been 7 %, $\frac{1}{6}$ of the above would have been
added to it. How much would have been added if the rate
had been 8 %? What would this interest have been? If
the rate had been 5 %, $\frac{1}{6}$ of the above would have been sub-
tracted from it. How much would have been subtracted if
the rate had been 4 %? What would the interest have been
at 4 %?

EXERCISES

Find the interests for the following by the bankers'
method given in Art. 98:

1. $ 4000 at 6 % for 2 yr. 2 mo. 10 da.

2. $ 460 at 6 % for 3 yr. 4 mo. 10 da.

3. $ 2400 at 6 % for 1 yr. 3 mo. 10 da.

4. $ 820 at 6 % for 1 yr. 5 mo. 20 da.

5. $ 3200 at 7 % for 1 yr. 4 mo. 20 da.

6. $ 1650 at 5 % for 2 yr. 3 mo. 5 da.

7. $ 2425 at 5 % for 1 yr. 7 mo. 10 da.

8. $ 1535 at 8 % for 8 mo. 25 da.

9. $ 2345 at 7 % for 2 yr. 5 mo. 15 da.

10. $ 834 at 8 % for 3 yr. 7 mo.

11. One of Mr. McClure's customers paid a debt of $ 86,
3 yr. 7 mo. after the debt was due. If Mr. McClure could
have loaned this money at 6 %, what should the customer
have paid him in addition to the original bill of $ 86?

12. Carry out: $(a + b)^2$; $(a - b)^2$; $(2 + c)^2$; $(3 - a)^3$;
$[2 + r]^2$; $\{1 + r\}^3$; $[1 + R]^3$; $(1 + .02)^2$.

99. Interest Tables.—Most men who compute interest a great deal, as loan agents and bankers, use interest tables such as the one given on page 69.   These tables are often larger so as to give the interest for any day from 1 up to 30 and for several years.   They also give the interests for various rates, as 2 %, 2½ %, 3 %, 3½ %, etc., up through 10 % or 12 %.

In using the table, place a small card or a straight piece of paper under the line containing the number wanted.   Do not rely upon tracing it out with a pencil or the finger.   Most errors in the use of tables are due to such tracings with a finger or a pencil.

Find the interest on $ 2350 at 6 % for 1 yr. 3 mo. 25 da.

Interest	$ 2000...... 1 yr. (10 × int. $ 200)...... =	$ 120.00
Interest	300...... 1 yr. .... =	18.00
Interest	50...... 1 yr. ($\frac{1}{10}$ of int. $ 500)...... =	3.00
Interest	2000...... 3 mo. (10 × int. $ 200) ..... =	30.00
Interest	300...... 3 mo. .... =	4.50
Interest	50...... 3 mo. ($\frac{1}{10}$ of int. $ 500)...... =	.75
Interest	2000......25 da. (10 × int. $ 200)...... =	8.33
Interest	300......25 da. .... =	1.25
Interest	50......25 da. ($\frac{1}{10}$ of int. $ 500) ...... =	.21
	Total......................	$186.04

EXERCISES

From the table on the opposite page find the interests at 6 % on the following loans:

1. $ 3800 for 4 mo.

2. $ 560 for 7 mo.

3. $ 3600 for 24 da.

4. $ 1250 for 8 mo.

5. $ 4850 for 19 da.

6. $ 2000 for 3 mo. 10 da.

7. $ 4500 for 2 yr. 5 mo.

8. $ 1500 for 1 yr. 15 da.

9. $ 340 for 3 yr. 2 mo. 10 da.

10. $ 6450 for 1 yr. 3 mo. 7 da.

6% INTEREST TABLE

Dᴀ.	$100	$200	$300	$400	$500	$600	$700	$800	$900	Dᴀ.
1	0.017	0.033	0.050	0.067	0.083	0.100	0.117	0.133	0.150	1
2	0.033	0.067	0.100	0.133	0.167	0.200	0.233	0.267	0.300	2
3	0.050	0.100	0.150	0.200	0.250	0.300	0.350	0.400	0.450	3
4	0.067	0.133	0.200	0.267	0.333	0.400	0.467	0.533	0.600	4
5	0.083	0.167	0.250	0.333	0.417	0.500	0.583	0.667	0.750	5
6	0.100	0.200	0.300	0.400	0.500	0.600	0.700	0.800	0.900	6
7	0.117	0.233	0.350	0.467	0.583	0.700	0.817	0.933	1.050	7
8	0.133	0.267	0.400	0.533	0.667	0.800	0.933	1.067	1.200	8
9	0.150	0.300	0.450	0.600	0.750	0.900	1.050	1.200	1.350	9
10	0.167	0.333	0.500	0.667	0.833	1.000	1.167	1.333	1.500	10
11	0.183	0.367	0.550	0.733	0.917	1.100	1.283	1.467	1.650	11
12	0.200	0.400	0.600	0.800	1.000	1.200	1.400	1.600	1.800	12
13	0.217	0.433	0.650	0.867	1.083	1.300	1.517	1.733	1.950	13
14	0.233	0.467	0.700	0.933	1.167	1.400	1.633	1.867	2.100	14
15	0.250	0.500	0.750	1.000	1.250	1.500	1.750	2.000	2.250	15
16	0.267	0.533	0.800	1.067	1.333	1.600	1.867	2.133	2.400	16
17	0.283	0.567	0.850	1.133	1.417	1.700	1.983	2.267	2.550	17
18	0.300	0.600	0.900	1.200	1.500	1.800	2.100	2.400	2.700	18
19	0.317	0.633	0.950	1.267	1.583	1.900	2.217	2.533	2.850	19
20	0.333	0.667	1.000	1.333	1.667	2.000	2.333	2.667	3.000	20
21	0.350	0.700	1.050	1.400	1.750	2.100	2.450	2.800	3.150	21
22	0.367	0.733	1.100	1.467	1.833	2.200	2.567	2.933	3.300	22
23	0.383	0.767	1.150	1.533	1.917	2.300	2.683	3.067	3.450	23
24	0.400	0.800	1.200	1.600	2.000	2.400	2.800	3.200	3.600	24
25	0.417	0.833	1.250	1.667	2.083	2.500	2.917	3.333	3.750	25
26	0.433	0.867	1.300	1.733	2.167	2.600	3.033	3.467	3.900	26
27	0.450	0.900	1.350	1.800	2.250	2.700	3.150	3.600	4.050	27
28	0.467	0.933	1.400	1.867	2.333	2.800	3.267	3.733	4.200	28
29	0.483	0.967	1.450	1.933	2.417	2.900	3.384	3.867	4.350	29

Mo.	$100	$200	$300	$400	$500	$600	$700	$800	$900	Mo.
1	0.500	1.000	1.500	2.000	2.500	3.000	3.500	4.000	4.500	1
2	1.000	2.000	3.000	4.000	5.000	6.000	7.000	8.000	9.000	2
3	1.500	3.000	4.500	6.000	7.500	9.000	10.500	12.000	13.500	3
4	2.000	4.000	6.000	8.000	10.000	12.000	14.000	16.000	18.000	4
5	2.500	5.000	7.500	10.000	12.500	15.000	17.500	20.000	22.500	5
6	3.000	6.000	9.000	12.000	15.000	18.000	21.000	24.000	27.000	6
7	3.500	7.000	10.500	14.000	17.500	21.000	24.500	28.000	31.500	7
8	4.000	8.000	12.000	16.000	20.000	24.000	28.000	32.000	36.000	8
9	4.500	9.000	13.500	18.000	22.500	27.000	31.500	36.000	40.500	9
10	5.000	10.000	15.000	20.000	25.000	30.000	35.000	40.000	45.000	10
11	5.500	11.000	16.500	22.000	27.500	33.000	38.500	44.000	49.500	11

Yʀ.	$100	$200	$300	$400	$500	$600	$700	$800	$900	Yʀ.
1	6.00	12.00	18.00	24.00	30.00	36.00	42.00	48.00	54.00	1
2	12.00	24.00	36.00	48.00	60.00	72.00	84.00	96.00	108.00	2
3	18.00	36.00	54.00	72.00	90.00	108.00	126.00	144.00	162.00	3
4	24.00	48.00	72.00	96.00	120.00	144.00	168.00	192.00	216.00	4
5	30.00	60.00	90.00	120.00	150.00	180.00	210.00	240.00	270.00	5

100. Compound Interest.—What is the interest on $ 1000 at 6 % for 3 yr. ? This is called **simple interest.** Suppose that the interest had been added to the principal at the end of the first year and that amount loaned the second year at 6 %; the interest the second year added to the principal and this amount loaned the third year at 6 %; what would the amount have been at the end of the third year ? Find this amount. When interest is computed in this way so that interest itself draws interest, it is called **compound interest.**

Referring to equation (4), page 61, we see that the amount at the end of the first year would have been $ 1000(1.06); at the end of the second year $ 1000(1.06)^2. Why ? At the end of the third year the amount would have been $ 1000 (1.06)^3. Why ? Use logarithms, which were studied in Book II—tables on pages 229–231—to find this. Hence, for any principal, rate, and years, when reckoning compound interest, we have

$$A = P(1 + R)^y. \qquad (1)$$

From (1) $I = A - P = P(1 + R)^y - P =$
$$P[(1 + R)^y - 1]. \qquad (2) \quad \text{How ?}$$

<center>**EXERCISES**</center>

1. Explain the above formulas fully.

2. Use the above formula to find the amount of $ 1000 placed at compound interest for 3 yr. at 5 %.

3. Find the amount of $ 600 at compound interest for 2 yr. at 4 %; at 6 %; at 5 %.

4. Find the compound interest at 5 % of $ 750 for 3 yr.

5. Find the amount of $ 100 at compound interest at 6 % in 10 yr.; 25 yr.; 50 yr.; 75 yr.; 100 yr.

6. Solve equation (1) above for *P*. Explain.

7. How much money placed at 6 % yearly compound interest will amount to $500 in 3 yr. ?

101. Compound Interest Table.—Tables are even more useful in finding compound than simple interest. The following table gives the amount of $ 1 compounded at the rates and for the time stated in years. How can the table be used if the interest is compounded semiannually ?

Yr.	2½ per cent	3 per cent	3½ per cent	4 per cent	5 per cent	6 per cent
1	1.025000	1.030000	1.035000	1.040000	1.050000	1.060000
2	1.050625	1.060900	1.071225	1.081600	1.102500	1.123600
3	1.076891	1.092727	1.108718	1.124864	1.157625	1.191016
4	1.103813	1.125509	1.147523	1.169859	1.215506	1.262477
5	1.131408	1.159274	1.187686	1.216653	1.276282	1.338226
6	1.159693	1.194052	1.229255	1.265319	1.340096	1.418519
7	1.188686	1.229874	1.272279	1.315932	1.407100	1.503630
8	1.218403	1.266770	1.316809	1.368569	1.477455	1.593848
9	1.248863	1.304773	1.362897	1.423312	1.551328	1.689479
10	1.280085	1.343916	1.410599	1.480244	1.628895	1.790848
11	1.312087	1.384234	1.459970	1.539454	1.710339	1.898299
12	1.344889	1.425761	1.511069	1.601032	1.795856	2.012197
13	1.378511	1.468534	1.563956	1.665074	1.885649	2.132928
14	1.412974	1.512590	1.618695	1.731676	1.979932	2.260904
15	1.448298	1.557967	1.675349	1.800944	2.078928	2.396558
16	1.484506	1.604706	1.733986	1.872981	2.182875	2.540352
17	1.521618	1.652848	1.794676	1.947901	2.292018	2.692773
18	1.559659	1.702433	1.857489	2.025817	2.406619	2.854339
19	1.598650	1.753506	1.922501	2.106849	2.526950	3.025600
20	1.638616	1.806111	1.989789	2.191123	2.653298	3.207136

EXERCISES

Use the above tables in the following:

1. Find the amount of $ 1000 compounded annually at 4 % for 6 yr.; for 13 yr.; for 17 yr.

2. Find the amount of $ 400 compounded annually at 3 % for 4 yr.; for 7 yr.; for 15 yr.

3. Find the amount of $ 1000 compounded annually at 3½ % for 3 yr.; for 8 yr.; for 10 yr.

4. Find the amount of $ 500 compounded annually at 5 % for 2 yr.; for 4 yr.; for 5 yr.

In most instances where compound interest is paid, as in savings-banks, the interest is paid twice yearly. The interest is said to be **compounded semiannually.**

5. Give the semiannual amount of $ 1 when the yearly rate is 6 %; 8 %; 12 %; 5 %; 4 %; 7 %.

6. Show that at 4 % interest compounded semiannually $ 100 in 5 yr. will amount to $ $100(1.02)^{10}$. Find this.

7. Write the equations for interest compounded semiannually corresponding to the equations on page 70.

**8.** Find the amount of $ 450 compounded semiannually at 4 % for 2 yr.; for 10 yr.; for 7 yr.

9. In 1626 Peter Minuit purchased Manhattan Island, a portion of New York City, from the Indians with goods valued at $ 24. If this sum had then been placed at 6 % compound interest, to what would it have grown to-day ?

10. The assessed valuation of Manhattan Island in 1917 was $ 5,088,344,403. Compare this with the growth of the original purchase price at 6 % compound interest.

11. Find the value of $ 100 left 9 yr. at 4 % interest, compounded semiannually.

12. When Henry was 6 yr. old his father placed $ 300 in the savings-bank for him. If this bank paid 4 % interest, compounded semiannually, how much can Henry draw from the bank when he goes to college at the age of 18 yr. ?

13. The war Baby Bonds were sold during January, 1919, for $ 4.24 each. They will be redeemed by the United States Government January 1, 1923, for $ 5.00. At 4 % interest, compounded semiannually, what will the $ 4.24 loaned January 1, 1919, amount to January 1, 1923 ? Compare *with the $ 5.00* paid by the government.
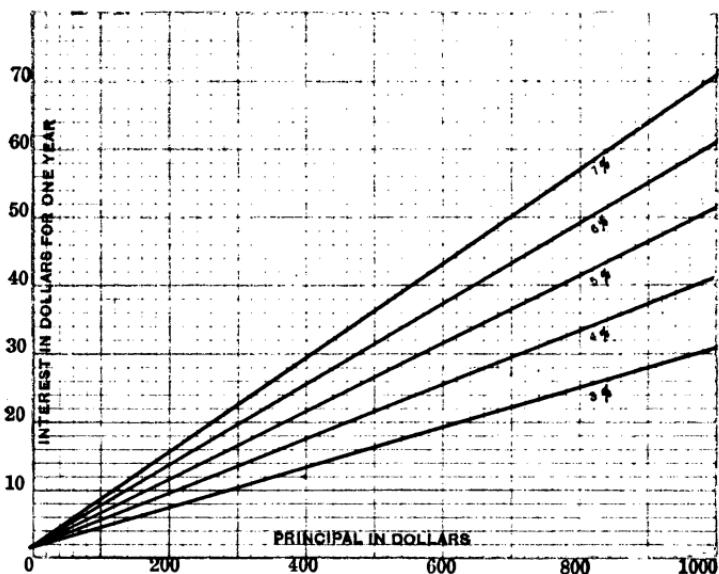
102. Interest Graph.—Interests can be approximated quite accurately by the above graph. Use the graph in solving the following exercises.

EXERCISES

1. Find the interest on $ 900 at 3 %; at 6 %; at 4 %.

2. Find the interest on $ 650 at 5 %; at 4 %; at 6 %.

3. What principal will produce $ 28 per year at 4 % ? at 5 % ? at 3 % ?

4. What principal will produce $ 35 per year at 7 % ? at 4 % ? at 6 % ?

5. Make an interest graph like the above for the same principals at 2 %; at 3½ %; at 4½ %.

6. Make up problems like Exs. 1–4 to be solved from your graphs. Verify all your results.

GRAPH COMPOUND AND SIMPLE INTEREST

Principal $ 100; rates 3 %, 4 %, 7 %
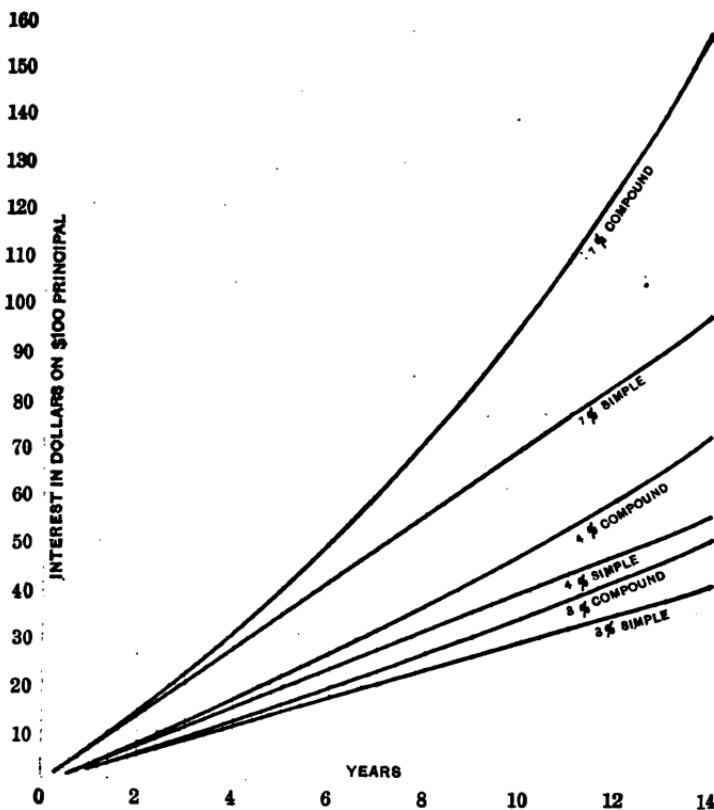
The above graph of simple and compound interests on a principal of $ 100 up to 15 yr. for varying rates is self-explanatory. It is especially instructive in showing the difference in earning power of money placed at compound and simple interests at the same rate. Note how some graphs go upward faster as the time increases. What does it mean to have the graphs go upward fast?

EXERCISES

1. From the opposite graph give the compound interest of $ 100 for 8 yr. at 3 %; of $ 100 for 14 yr. at 7 %.

2. From the opposite graph give the compound interest of $ 100 for 5 yr. at 7 %; of $ 100 for 13 yr. at 4 %.

3. From the opposite graph find how long it will take $ 100 to earn $ 70 at 7 % compound interest.

4. How long will it take $ 100 to earn $ 70 at 4 % compound interest ?

5. How long will it take $ 100 to earn $ 80 at 7 % simple interest ? to earn $ 100 at 7 % compound interest ?

6. When John began school at 6 yr. of age his father placed $ 100 in the savings-bank at 4 % compound interest. How much will John have when he is 18 yr. to help him through college ?

7. Study the interest graphs on the opposite page very carefully and state what important facts they tell.

8. By use of the interest table found on page 71 make a compound interest graph for the first 10 yr. or the first 15 yr. in which the principal is $ 100 and the rate 5 %. On the same co-ordinate axes also draw the simple interest graph.

9. Suggest questions similar to Exs. 1–5 for your graph and answer these questions.

10. From the graph on the opposite page find the compound interest of $ 300 at 4 % for 14 yr.

11. Find the compound interest of $ 250 at 4 % for 10 yr.

12. At what rate of interest, compounded annually, will $ 800 amount to $ 882 in two years ? Substitute in (1) on page 70 and solve for R.

103. Promissory Notes.—If Mr. Rowe borrows $ 1000 from Mr. Doe, he gives Mr. Doe a **promissory note,** usually called a **note.** Mr. Rowe is the **maker,** Mr. Doe the **payee,** and $ 1000 the **face** of the note. The **date of maturity** is the time when the note is due.

$1000.00   Monroe, Ala. Jan. 16, 1919
60 days _____ after date I promise to pay to
the order of   John Doe
One thousand and no/100 _____ Dollars
at Planters Bank, Monroe, Ala.
Value received with interest, at 8 per cent per annum
No. 27   Due mar. 17, 1919   William R. Rowe

When two or more men borrow money for their business, it is customary to make out a note as below. Here each man binds himself not only to pay his share but to pay all if the others cannot pay their shares.

No. 97 _____   March 17 1920
ninety days after date we jointly and severally as principal
promise to pay to the order of Jacques Q. Lugues
the sum of nine hundred fifty + no/100 Dollars
With interest at the rate of 7 per cent per annum from
date Value received
Payable at Commercial Bank   John J. Doe
Post Office St. Paul, Minn.   Richard J. Rowe
$950.00

The statement "Value received" is not necessary to make a note legal, but shows the good faith of the transaction that led to the making of the note.

A **demand note**, as below, is due whenever payment is demanded.

Each of the illustrations given contains the words "order of." Such notes are said to be **negotiable**; that is, they can be sold by the owner. If the words "order of" were cancelled out the note could not be transferred to a third party. When a note is sold it is **indorsed** by the owner of it—see page 79.

Notes that call for no interest do draw interest after becoming due, and demand notes, after payment has been demanded also draw interest. The rate is the **legal rate**—see page 235.

In writing practice papers, as notes, checks, and so on, never use your real name. Why not?

EXERCISES

1. Who was the maker, the payer, and the payee in each of the three notes shown?

2. How could each note have been made non-negotiable?

3. What is the value of the first two notes when due?

4. As a rule it is good business policy not to make demand notes. Why?

5. Write a note on $ 650, due in 60 da., interest 6 %, made by John R. Jackes payable to Henry Q. Sonz.

6. Write a demand note.

104. Coupon Notes.—A common form of note for long-term loans, as three years or longer, is the coupon note. The note proper.is made out very much like either of the notes on page 76. Attached to the note are coupons for each interest, which are detached and presented to the maker of the note when the corresponding interest is paid. These coupons are themselves notes and each begins to draw interest from the day that it should have been redeemed by payment of the interest on the original note. A $2000 coupon note for a term of 5 years drawing interest at 6 % to be paid annually has attached to it five coupon notes due 1, 2, 3, 4, and 5 years after the note was written. If the coupon notes draw 8 % interest and any one of them is not redeemed before 3 mo. after the interest is due, the payment will be not $120 but an additional 8 % on the $120 for 3 mo. What will this be?

EXERCISES

1. Write a 90-day note bearing 6 % interest on $650, with yourself as payee and John X. Janks as maker. Compute the amount at maturity (when due).

2. Write a demand note on $500 at 7 % with Henry Y. Hork as maker and yourself as payee. What is the amount after 20 da.? after 45 da.? after 60 da.?

3. Secure a blank form of a coupon note from a bank or stationery store. Study and explain the form to the class.

4. Fill in a coupon note on $3500 at $5\frac{1}{5}$ % interest due in 3 yr., with interest payable annually. The payee is Jarus Z. Hone and the maker Jules V. Ruso.

5. If the first interest is paid 4 mo. after it is due, what will be the payment?

105. Indorsements. — There are four usual forms of indorsement, which are here illustrated.

(1) Indorsement in blank. Here the indorser promises to pay the note if the maker does not do so. It is customary to require such an indorsement by the payee of a note that he sells. Such indorsements are also made as a guarantee that the maker will pay the note when due.

(2) Special indorsement. Usually made by the payee indicating that the note is to be paid to the one named.

(3) Qualified indorsement. The one buying such a note agrees to accept it without holding the payee responsible for its payment.

(4) Payment indorsements. It is also customary to record upon the back of a note all payments of either interest or parts of the principal that have been made.

Henry Dowe

No. 1

Pay John Rowe
Henry Dowe

No. 2

Pay John Rowe
without recourse
Henry Dowe

No. 3

Jan. 14, 19 rec $35
Sept. 25, 19 rec. $85

No. 4
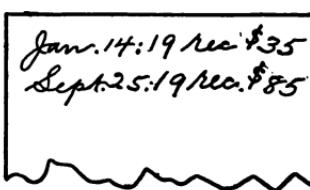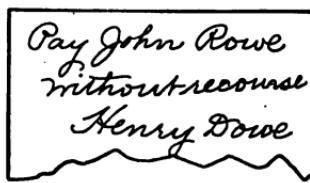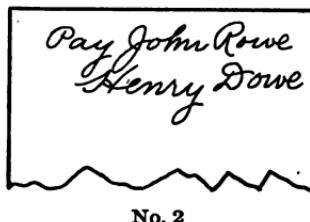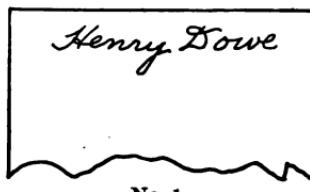
EXERCISES

1. Indorse the note for Ex. 1, on page 78, to Wm. S. Muchmoney, without recourse.

2. Indorse the note for Ex. 2, on page 78, by Rayo Q. Bouse as a guarantee of payment by maker.

3. Indorse the note of Ex. 3, on page 78, to Hemus U. Lender, making the payee also liable for the payment.

106. Bank Discount.—It is the custom of most banks to collect interest on a loan when it is made, instead of when it is paid. Interest collected thus in advance is called **bank discount.** Notes made for such loans as a rule call for the payment of the principal without interest. Thus, any one borrowing $ 1000 at a bank at 8 % interest for 90 da. would make out a note promising to pay the bank $ 1000 after 90 da. without interest. The bank then deducts the interest, $ 20, and pays over to the borrower $ 980, called the **net proceeds.**

Business men also discount—sell—notes that they hold, made in favor of themselves. Suppose that Mr. Wilkes had taken a note from Mr. Jilson for $ 2000 at 6 % payable at the end of a year. If Mr. Wilkes needs the money after holding the note 9 mo., he may discount it at his bank. The future value of the note, that is, the amount when due, is first computed, which here is $ 2120. How? This will be the amount paid the bank, just as the $ 1000 in the case first mentioned. The interest on $ 2120 for the remaining 3 mo. at the discount rate, say 8 %, is now computed and deducted from $ 2120 to give the net proceeds. What is the interest on $ 2120 at 8 % for 3 mo. ? What are the net proceeds ?

EXERCISES

1. Find the net proceeds of a 60-da. note on $ 800 discounted at the bank at 8 %.

2. Find the net proceeds of a 90-da. note on $ 650 discounted at the bank at 10 %.

3. Find the net proceeds of a 30-da. note on $ 1750 discounted at the bank at 8 %.

4. State by an equation that the proceeds, P, equal the principal, p, less the discount, d. Express P in terms of p, r, and t. Solve for p. What does this say ?

5. Write a note for the loan mentioned in Ex. 1, page 68, making yourself as payee and Henry X. Doe as maker. What will be the net proceeds if discounted 2 mo. before it is due at 8 % ?

6. Write a note for Ex. 1, page 68. Find the net proceeds if discounted 2 mo. before it is due at 10 %.

7. Write a note for Ex. 2, page 68. Find the net proceeds if discounted 3 mo. before it is due at 8 %.

8. Write a note for Ex. 4, page 68. Find the net proceeds if discounted 4 mo. before it is due at 8 %.

9. Mr. Wilson has a 6-mo. note made out in his favor 5 mo. ago for $ 500 at 7 %. What will be the net proceeds if he discounts the note at the bank at 8 % ?

107. Mortgages.—Loans of several hundred dollars, especially those taken for a long time, as 3 yr. or 5 yr., are usually secured by property of some form. This security may be **real estate,** as land and buildings, or it may be **chattel,** as a stock of goods, live stock, machinery, and so on. The one borrowing the money makes out a statement specifying what property is to secure the payment of a certain note. This statement is called a **mortgage** and the note a **mortgage note.** In order to secure himself against loss or fraud the holder of a mortgage has it recorded at the county office of register of deeds in some states and at the office of the county clerk in others. If the maker of a mortgage note cannot pay it when due, the property that secured it is sold and the holder of the note paid from the sale, if these funds are sufficient. Any amount of the note not thus paid is still owed by its maker. Any amount realized from such sale above the indebtedness goes, of course, to the maker of the note.

108.   Bonds.—Governments, states, counties, cities, and so on, together with large establishments, as railroads and manufactories, very often borrow large sums of money. These vast sums are obtained from very many persons, to whom notes called **bonds** are issued.   If the money is borrowed by a firm, the bonds are secured by a mortgage, either on a part or on all of its property.   Bonds differ from ordinary notes in that they are bought and sold, rise and fall in value, as any other property.   Bonds are usually sold through a broker, as will be explained in Chapter XIII.

The amount stated upon a bond is called its **face value,** or its **par value.**   Face values vary, but are generally $ 1000.

Usually bonds sell for either more or less than their face values.   If a $ 1000 bond sells for $ 870 it is said to sell for $ 130 **below par.**   If a $ 1000 bond sells for $ 1350 it is said to sell for $ 350 **above par.**   In the former case the bond would sell at 87 % of the par value, or 13 % below par.   For what per cent of par and what per cent above par would it sell in the second case ?   Look in the daily papers for quotations on bonds.   If the one issuing the bonds has a high financial rating and if the rate of interest is good, bonds will likely sell above par.   Why ?

Interest on bonds is always computed upon the face value, whether they sell above or below par.   A 4 % bond of face value $ 1000 will give a yearly return of $ 40.   Why ?

109.   Registered Bonds.—There are two general forms of bonds, **registered** and **coupon.**   A registered bond, like the one shown on page 83, has the name of the owner stated either on the face or on the back of it.   The name of the owner is also entered upon the books of the concern issuing the bond.   Ownership of a registered bond depends upon the

name in the books of the concern issuing it and not upon possession of the bond; hence, loss of a bond is of little consequence. Interest on registered bonds is paid by check.

110. Coupon Bonds.—Coupon bonds, like the one shown above, contain coupons for each interest just as the common coupon notes described on page 78. The coupons when due are often left as deposits at a bank which collects them for its customers. Coupon bonds can be sold as any other property without notifying the concern issuing them. Therefore, while it is much simpler to trade in coupon bonds, they are not nearly as safe as registered bonds. Why? Coupon bonds should be kept in a safe place, as well as a description of their numbers in a different place. Why?

EXERCISES

1. How much above or below par is a $ 1000 bond bought for $ 945 ? for $ 1215 ? for $ 765 ?

2. What will a school district receive for each $500 bond sold at 98 % of its par value ? at 104 % par value ?

3. What is the yearly income from a $ 1000 city bond drawing 5 % interest ? from eight such bonds ?

4. What is the yearly interest from six $4\frac{1}{4}$ % U. S. bonds, par value $ 100 each ?

5. What is the yearly income from seven $4\frac{1}{4}$ % U. S. bonds, par value $ 500 each ?

6. What will be the yearly income from a $ 1000 bond drawing 6 % interest issued by the Grange Elevator Co. ? Suppose that this bond was bought for $ 1145, how much was invested to get a return of $ 60 per year ? What rate of interest was this on the $ 1145 invested ?

7. What is the rate of investment of a $ 500 city bond drawing 5 % interest if bought at $ 435 ? at $ 540 ?

8. What is the yearly income of a $1000 Pike township 5 % interest bond ? If the usual rate of interest is 7 %, how much can you afford to pay for it ?

9. What is the yearly income from a $ 1000, r % bond ? State by an equation what should be paid for such a bond so as to make R % on the money invested.

10. For what must a 6 % $ 1000 county bond be bought so that it will pay 5 % interest ? 7 % interest ?

11. For what must a 5 % $ 1000 state bond be bought so that it will pay 6 % interest ? 4 % interest ?

12. Would you rather own a registered or a coupon bond issued by the same company for the same amounts and drawing the same rate of interest ? Why ?

111. Savings.—All young persons should lay aside for investment at least a part of what they earn or have given to them. In no other way can they become independent. Ownership of a safe investment is one requirement for membership in the "Don't Worry Club."

Suppose that you are able to lay aside P dollars during the year and invest it the first of the next year at $R \%$ interest, and suppose that this is carried out for n years, then the sum accumulated by the end of these years is found by higher literal numbers to be

$$S = P\,\frac{[(1 + R)^n - 1]}{R}.$$

Use logarithms in computing by this formula.

If a man can save only $2 a week, or $104 a year, what will he accumulate at 5 % interest in 10 yr. ? in 25 yr. ?

EXERCISES

1. Suppose that Henry, who carries papers, saves 50 ¢ of his earnings each week, how much will he save in a year? By the use of the above formula find how much this will amount to if he keeps up the same savings, placing it at the end of each year at 4 % interest for 5 yr.; for 8 yr.

2. What will $300 invested yearly at 6 % become in 5 yr. ? in 10 yr. ? in 15 yr. ?

3. Henry's father puts $3 in the building and loan association each week. At the end of each year he is paid the amount put into the association and $4 interest. This he invests in bonds that pay 5 % interest compounded annually. How much will he accumulate in 5 yr. ? in 10 yr. ?

4. What will a saving of $5 per week for a year amount to ? If this is placed at 4 % compound interest each year what will the amount be in 5 yr. ? in 10 yr. ? in 25 yr. ?

112. What is a Good Investment?—Every investment must be looked at from two standpoints, **security** and **rate of interest.** Security must always be considered first. . It matters not how high the rate of interest may be; if the security is poor, then it is a poor investment. Too many have been led astray with the prospects of large returns and have lost both principal and earnings. The following are a few rules for the inexperienced investor:

1. Investment in any untried venture is risky.

2. As a rule no one will sell a thing this week for less than he knows that he can sell it next week.

3. One who offers a rate of interest higher than the usual one does not have good security to offer.

4. Invest, but never speculate.

5. Never accept a stranger's statement regarding any security; nor the statement of a friend who is not an expert in the particular enterprise in question.

6. Always get the advice of *your banker* on any investments you contemplate. *The bankers* of your city are safe, unbiassed expert advisers on investments.

7. Make your slogan: "Safety first, rate of interest second."

EXERCISES

1. What is the first thing to look at in making an investment? Apply this to buying mining-stock.

2. Mr. Wools owns a farm valued at \$ 20,000, upon which he borrowed \$ 3000 at $5\frac{1}{2}$ % interest. Another man borrowed \$ 6000 on a farm valued at \$ 8000 and paid $7\frac{1}{2}$ % interest. Why the difference in rate of interest?

3. What place should rate of interest take in making an investment?.

1

4. A man bought some city lots for $ 2500, on which he paid a yearly tax of $ 45. If he could have invested the money at 6 % interest, what did the lots cost him at the end of the year ? at the end of 2 yr. ? at the end of 5 yr. ?

5. What is the net yearly income from a house that rents for $ q per month where the insurance is $ I, taxes $ T, and other expenses $ E ?

6. If the house in Ex. 6 is valued at $ D, what rate of interest does it bring on the money invested ?

7. A house can be bought for $ 4000, that will rent for $ 32 per month. Taxes are $ 54, repairs $ 65, and insurance $ 14 per year. What will be the net income after these payments ? Will this be more or less than 6 % interest for the money invested ? Is it a good investment ?

113. Partial Payments.—It often happens that notes are not paid all at once. Why ? Each payment is indorsed upon the note, as mentioned on page 79.

Two rules are used to compute the **present worth**—value —of a note upon which payments have been made. One is to find the amount at the time of the first payment and from this to subtract the first payment to get a new principal. This is then repeated for every payment. There is a United States law that makes this illegal if the payment is less than the interest up to the time of payment. Thus, suppose that a note for a year at 6 % on $ 3000 has a payment of $ 50, made 10 mo. after the note was written. The interest is $ 150 up to the time of payment of the $ 50. Hence, this $ 50 is merely indorsed upon the note and subtracted when the note is paid. Show that subtracting the $ 50 from the amount of the note at the time it is paid would be to charge compound interest for the remaining two months of the note.

The other rule is to find the amount of the note at final settlement as if no payments had been made. From this amount at final settlement is subtracted the amount of each payment as if it had been placed at interest until the time of payment. For the above the amount at the end of the year is $ 3180, from which is subtracted the amount of $ 50 for 2 mo., or $ 50.50.

EXERCISES

1. A note for 1 yr. at 8 % on $ 2000 had a payment of $ 500 made 3 mo. before it was due; what should the maker of the note pay at maturity ?

2. What is the value at maturity of a 6-mo. note at 6 % on $ 700, if a payment of $ 300 was made 4 mo. before due ?

3. A note for $ 450 dated Feb. 5 and due in 6 mo. at 8 % had the following payments made upon it: $ 200 when due; $ 150, 2 mo. after due. What should the maker pay in settling the debt 1 yr. after the note was made ?

4. Write a note for the above with yourself as payee and John Doe as maker. Make the proper indorsements.

5. A note dated Oct. 8 and due Jan. 6 on $ 800 at 5 % had a payment of $ 350 made Dec. 7. What is the value of the note at maturity ?

6. Mr. Jaquath bought a farm and gave a 6 % mortgage note on $ 3700 for 5 yr. as part payment. He had the privilege of paying $ 100 or any multiple thereof when he paid the yearly interest. Besides the interest he paid $ 100 the first, $ 300 the second, $ 500 the third, and $ 200 the fourth year. What did he save in simple interest by these payments ? What did he pay when the note was due ?

7. Henry saved $ a, $ b, and $ c for each of 3 mo. respectively. What did his savings average per month ?

8. How much would Henry save per year at the above average ? How much interest would this money earn at 5 % ? at r % ?

114. Buying on Payments.—More is usually paid for anything bought on the instalment plan. Property is sometimes paid for in equal instalments. Thus, a house may be purchased for $2500 and paid for in 10 equal yearly instalments. How much will these equal amounts be? A town may also pay a debt by equal extra taxes for a stated number of years. By higher literal numbers it has been computed that p, in the following formula, gives the equal amounts in which R is the rate of interest to be charged, n the number of years or equal payments, and A the price or amount to be raised:

$$p = \frac{A(1+R)^{n-1}}{(1+R)^n - 1}.$$

Building and loan associations are established in many cities and towns. These accept small amounts for investment upon which they pay compound interest. They also loan money upon real estate which is usually paid for in monthly or yearly instalments.

EXERCISES

1. Mr. Bliss bought a house for $4500 on the following terms: $1200 cash and the balance in 10 equal yearly instalments at 5 % interest. What was each payment?

2. Miss Martin bought a piano for $350. She paid for it in 5 yearly payments of $90 each. Allowing interest at 6 %, were the payments correct?

3. What will be the amount of 5 equal yearly payments for a lot costing $800, if money earns 6 % interest?

4. A man bought a house and lot costing $2250 with money furnished to him by a building and loan association. If interest is 6 %, what yearly payments should he make to pay for it in 10 yr.? in 15 yr.? in 7 yr.?

VII

BANKING

115. Functions of Banks.—Banking had its origin with the money-exchangers of old who stationed themselves in the market-places to change money for the purchasers. Gradually these exchangers began to accept money for safe-keeping and to lend money. From this crude origin among the ancients has come our modern bank.

The principal functions of our banks of to-day are to accept money on deposit for safe-keeping; to provide a safe, simple medium for transferring money, as in paying a debt; and to lend money. National banks may also issue bills—paper money—which are secured by government bonds deposited with the comptroller of the currency at Washington. Notice some money in bills to find one issued by a bank and not by the United States Government.

116. Opening an Account.—In opening an account at a bank your signature is first taken and filed for identification if needed. How could such a need arise? Some banks require references of strangers opening a new account. Why? A bank may also require a certain average deposit or ask a small charge of the depositor, as one dollar per month, before it will carry a checking account. This is especially true of the banks in the larger cities. The average daily deposit required varies from $ 50 to $ 500.

117.   Deposits.—It is the duty of depositors to fill out a deposit slip such as is here shown every time a deposit is made. It is the privilege of the bank to have a record of your deposits in your handwriting. Why? Before turning in your deposits place currencies of like denominations together and indorse all drafts and checks properly. (See page 79.) After receiving your bank-book from the receiving teller, verify the amount he recorded. While the bank credits you with checks, it accepts these only as your agent and assumes no responsibility. Verify the above deposit slip.

DEPOSITED BY

Henry L. McGuire

IN THE

Citizens National Bank

EVERETT, WASHINGTON.

Oct. 9,   191 *9*

	Dollars	Cents	Dollars	Cents
Bills	125			
Silver	29			
Gold				
Check	36	42		
"	9	87		
"	124	08		
"	79	65		
"	402	02		
"				
"				
"				

EXERCISES

Make out deposit slips for the following:

1.   M. T. Jacks: silver, $ 34.64; currency, $ 205; checks, $ 364.62; $ 23.19; $ 35.08.

2.   A. K. Mummer: silver, $ 15.24; currency, $ 35.00; checks, $ 8.95; $ 45.82; $ 23.08; $ 19.47; $ 28.76.

3.   Q. P. Olus: silver, $ 74.93; currency, $ 450; checks, $ 74.37; $ 108.47; $ 216.84; $ 85.32; $ 28.73; $ 132.47.

4.   Name three uses that you may make of your bank.

118. Checks.—John V. Doe may cash the above check, deposit it at his bank, turn it over as cash to some one, or use it in payment of a debt. In any case the check must be indorsed by John V. Doe; hence, is worthless to any one else unless indorsed. If John V. Doe loses the check, he should report that fact at once to Mr. Roakes and to the bank upon which it is drawn, to have payment stopped. In depositing or cashing the check it need be indorsed merely by the name John V. Doe. If the check is turned over to a third party, it should be indorsed as here shown. It must then be indorsed by Henry Z. Jones before the bank will cash it. Why is this form of indorsement a protection to Mr. Jones? Note that the indorsement was made on the left end of the check.

If the words "order of" had been omitted—a line drawn through them—the check could have been cashed by no one but John V. Doe.

No. 673   DES MOINES, IOWA _Feb. 25th_   19 20

The Corn State Bank

Pay to _John M. Doe_ _____ or bearer $ 19 37/100

nineteen & 37 _____ Dollars

L. R. Vokes.
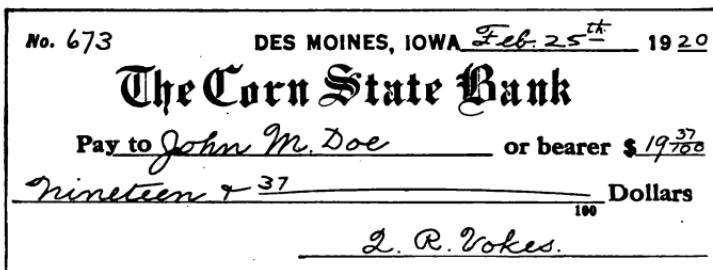
The above check can be cashed by any one. Why? Is it as safe as a check made "to order"? Banks in some communities, however, require such checks to be indorsed before accepting them. Checks are also made payable to "self," when only the maker can cash it. A check made payable to "cash" is payable to any one without indorsement.

In order to avoid having a check raised, begin the numerals close to the dollar-mark; also begin the statement in words close to the left-hand margin and draw a line to the word **Dollars,** if necessary. Criticise the illustrations. Banks protect their drafts and many large establishments protect their checks by stamping in the amounts in such a way as to perforate the paper.

After writing each check fill in the stub as here shown. Do not depend upon your balance at the bank, as checks you have drawn may not have been cashed. Deposit without delay all checks drawn in your favor. How may this save the makers of these checks inconvenience?

	DOLLS.	CTS.
BAL.	236	16
DEP.		
TOTAL	.	
THIS CHECK	19	37
BAL. FWD.	216	79

Make all checks payable to order and preserve them after they have passed through the bank, as possible receipts for the bills which they have paid. Such checks make reliable receipts because they must be signed by the person or firm to whose order they were drawn.

Never accept a check from a stranger without identification. As you must indorse it before having it accepted by your bank or by any other person, you make yourself responsible for its payment. If you lose a check made out in your favor or if a check you made out in some other person's favor is lost, at once notify the bank upon which it is drawn to stop payment. Then have a duplicate check issued.

EXERCISES

1. Write a check payable to J. Z. Sones or order for $ 45.32 on National Corn Bank and sign it John Doe.

2. Properly indorse this check, making it payable to Q. R. Misth or order.

3. Write a check payable to the order of Royal Manufacturing Co. for $ 235.50 on the City State Bank, and sign it Joe Roe.

4. Properly indorse the above check, making it payable to the Sweet Grocery Co. or order.

5. Make out a check for $ 10, payable to yourself on your own bank, the Royal State Bank.

6. Mr. Roote gave a check for $ 15, payable to the order of Mr. Deas, who lost it. It was found by a tramp who forged Mr. Deas's indorsement to the check and presented it to the Emporium Clothing Co. in payment for a cap. The clothing company accepted the check and paid the tramp the difference between the cost of the cap and the $ 15. Who had to stand the loss?

7. Explain how a check can be used as a receipt.

IN ACCOUNT WITH
THE CITIZENS NATIONAL BANK
JACKSONVILLE, MISS.

NAME_____Wm. Q. Sonz,_____

ADDRESS_____3516 Market St._____

CHECKS IN DETAIL		DATE 1919	DEPOSITS	BALANCE
BALANCE BROUGHT FORWARD ☞		May 1	139.67 .	.
9.45 – 13.62		2		116.60
27.48 – 86.29		5	219.13	221.96
19.74 – 8.56		8	109.38	303.04
108.57 – 47.06		10		147.41
42.18		12	93.75	198.98
79.50 – 4.82 – 19.68		19		94.98
19.25 – 36.40		24	205.60	244.93
16.95 – 47.38 – 35.45		27		145.15

119. Customer's Ledger Page.—The above page from a bank's ledger is self-explanatory. The last column to the right gives the balance at the end of the date at the left. Many banks use loose-leaf ledgers, and when an account is balanced deliver that leaf over to the depositor. The last number in the last column to the right is then the balance. Study the above page and explain fully.

120. Time Deposits.—So far we have studied only **open or checking accounts.** As a rule banks do not pay interest on such accounts. Money may, however, be placed at most banks for a definite time when interest will be paid on the same by the bank. Such deposits are usually for 3 mo. or 6 mo. and are called **time deposits.** The depositor receives a certificate of deposit from the bank. No checks can, of course, be drawn against such an account.

121. Daily Balance.—Firms and institutions, as a city or a state treasury, often have large continued bank balances upon which banks pay a low rate of interest. Interest is computed upon the average **daily balance** and then only for exact hundreds of dollars. To the right is shown a daily balance sheet. Adding these amounts shows that $ 79,000 had been on **deposit for 1 month.** How ? What interest, at $2\frac{1}{2}$ %, should be added to the account for the month ?

1	56,319.24	16	83,819.42
2	78,248.19	17	79,606.75
3	63,485.17	18	79,606.75
4	63,485.17	19	87,008.09
5	84,237.05	20	84,168.59
6	93,546.63	21	83,705.64
7	89,736.48	22	79,320.17
8	83,642.57	23	75,428.18
9	87,510.19	24	74,609.13
10	91,307.60	25	74,609.13
11	91,307.60	26	71,815.26
12	89,763.11	27	69,718.03
13	84,809.70	28	70,163.17
14	85,717.93	29	68,234.80
15	80,054.25	30	65,016.35

EXERCISES

For Exs. 1 and 2 make out the ledger account, the daily balances, and find the interest at 2 % :

1. Balance: $5683.49. Deposits: Jan. 7, $ 856.17; Jan. 11, $ 6145.32; Jan. 17, $ 406.72; Jan. 25, $ 3051.45. Checks: Jan. 4, $ 403.71; Jan. 10, $ 2031.48; Jan. 11, $ 45.65; Jan. 14, $ 45.08; Jan. 17, $ 131.60; Jan. 24, $ 405.93.

2. Balance: $ 6423.54. Deposits: Feb. 6, $ 4051.32; Feb. 11, $ 2036.17; Feb. 18, $ 2307.18; Feb. 23, $ 736.27; Feb. 27, $ 3006.37. Checks: Feb. 3, $ 206.83; Feb. 7, $ 87.96; Feb. 11, $ 405.73; Feb. 17, $ 208.19; Feb. 20, $ 506.73; Feb. 24, $ 218.67; Feb. 27, $ 2008.79; Feb. 28, $ 154.97.

3. What will be the amount of $ 350 if placed in a bank on a 3-mo. time deposit at 4 % interest ? at $3\frac{1}{2}$ % interest ?

122. **Clearing-House.**—Mr. *A*. pays a debt of $ 50 to Mr. *B*. by a check upon the First National Bank of their city.   Mr. *B*. deposits the check in the Second National Bank, which credits Mr. *B*.'s account $ 50.   To complete the transaction the Second National Bank takes the check to the First National Bank and receives $ 50 for the same. The First National Bank then debits the account of Mr. *A*. $ 50.

In a city with several banks it is impracticable for each bank to go to each of the other banks with the checks it has paid.   To facilitate these interchanges of checks the banks form what is known as a **Clearing-House Association.** Without going into details the workings of a clearing-house are as follows: Each bank of the association sends daily to the clearing-house the checks it received the previous day.   For these it receives credit; it has paid out money.   Then it— and every other bank—is debited with each check made out upon it and paid by another bank; another has paid out money for it.   If bank *N*. has brought checks to the clearing-house to the value of $ 56,345.83 and the other banks have brought in checks that they have paid drawn upon bank *N*. to the value of $ 48,707.29, bank *N*. has paid out $ 7638.54 more for the other banks than they have paid out for it. The other banks must pay bank *N*. that much to balance the day's transactions.   In practice, however, only the balance is kept from day to day and a money settlement made weekly, or every two weeks.   Each bank bears its share of the expense.

In a city with few banks one bank usually acts as the clearing-house, or each bank acts a month in its turn.

To facilitate the handling of checks received upon banks in other cities—**foreign checks,** so called—each bank has a *representative* bank in some of the large business centres,

as New York and Chicago. The banks in your city have deposits in what cities? Suppose a merchant in Waco, Texas, sent a check to a manufacturing company at Elgin, Ill. The Elgin bank at which the check is deposited sends the same to its representative bank at Chicago. The latter credits the Elgin bank with the amount of the check and sends it to the Chicago Clearing-House. From here it goes to the bank representing the Waco bank, which debits the Waco bank the amount of the check and forwards it to the Waco bank. When the Waco bank receives the check, the maker is debited its amount and it is returned to him when his account is balanced.

123. Protested Checks.—A check drawn by a person having insufficient funds goes back through the clearing-house to the person depositing it. Trace its route. All who have indorsed the check are notified by a notary public, which action is called **protesting**. This cost is added to the amount of the check. A protested check serves as a note.

EXERCISES

1. The financial part of a daily paper stated that for a certain week the bank clearance was $3,546,377,000 for New York; $438,079,000 for Chicago; $315,219,000 for Boston. Explain what this means. If necessary, ask a banker.

2. A certain city has five banks: A, B, C, D, and E. Find the debit or the credit balance for each bank if they turn the following checks into the Clearing-House one day:

A on B $3405.74, ...C $1238.52, ...D $ 968.35, ...E $2004.86;
B on A $1782.09, ...C $3007.68, ...D $3045.78, ...E $1708.34;
C on A $2045.70, ...B $1560.67, ...D $ 978.34, ...E $2340.67;
D on A $2306.48, ...B $ 987.15, ...C $3400.78, ...E $1467.09;
E on A $2308.45, ...B $1809.56, ...C $2406.74, ...D $2560.48.

☞BRING THIS BOOK WITH YOU

Date	Withdrawals		Deposits		Int.		Balance	
Aug. 1			50	–				
" 15		.	30	–			80	–
Sep. 3	20	–			.		60	–
Oct. 7			10	–			70	–
Nov 9			40	–			110	–
Jan. 1					?	?	?	?

124. Savings-Banks.—Savings-banks are state banks. However, most national and state banks maintain savings departments. A depositor in a savings department cannot check against it, but must bring his bank-book each time he wishes to make a withdrawal. As a rule, withdrawals are permitted at any time, but the banks reserve the right to require 30-da. or 60-da. notices of withdrawals. These rights they exercise only in times of money stringency.

Interest is paid on savings deposits at a low rate, usually 4 % or less. In most communities the interest is paid semi-annually, but in some it is paid quarterly. If the interest is not collected on interest-paying dates, it will be added to the principal at those times. Interest is thus compounded semiannually. Explain.

The time and the amount upon which interest is paid varies for different communities. The following practice is quite common, however: no interest is paid on fractional parts of a dollar. Upon how much would interest then be paid on a deposit of $ 105.87 ? No interest is paid on money

WHEN YOU WISH TO DRAW MONEY

Date		Withdrawals		Deposits		Int.		Balance	
Mar. 1				40	—				
Apr. 3				30	—			70	—
May 5				20	—			90	—
May 27		30	—					60	—
June 1				10	—			70	—
July 1						?	?	?	?

on deposit a fractional part of a month. Money on deposit for $3\frac{1}{2}$ mo. would then draw interest for how long? Interest is paid only on money on deposit at interest-paying dates. Withdrawal is made from the *amount longest on deposit*.

Suppose, in a savings-bank paying interest Jan. 1 and July 1, that $ 85 was deposited Jan. 20 and $ 40 May 8, and that $ 30 was withdrawn May 23. The $ 30 would be deducted from the $ 85 leaving $ 55 for which interest will be paid 5 mo. and $ 40 for 1 mo; or $ 315 for 1 mo. How?

EXERCISES

1. Compute the interest for the account on page 100 due Jan. 1, the next interest-paying date, at 3 %.

2. What is the balance of this account on Jan. 1?

3. Complete the above page by computing the interest due July 1, at $3\frac{1}{2}$ %, and by footing up the balance.

4. Make up a ledger page for the following items: Deposits: Jan. 1, $ 45; Feb. 18, $ 19.50; Mar. 16, $ 20.45. Withdrawals: Mar. 1, $ 10; Apr. 12, $ 15; May 7, $ 20.

125. Postal Savings Banks.—In the **postal savings department** the U. S. Post Offices accept for deposit amounts of $ 1 and up to a maximum of $ 100 for any one month. The most that any one person may have on deposit is $ 500. These deposits bear 2 % interest if left for one year. They begin drawing interest the first day of the month following the one in which the deposit was made. Money so deposited may be withdrawn by surrendering the certificate of deposit at the post office where deposit was made. Certificates of deposit may be changed into U. S. government registered or coupon bonds bearing 2½ % interest Jan. 1 and July 1. An amount of $ 500 of these bonds may be held by any one person in addition to $ 500 of certificates of deposit.

Since the issuing of Thrift Stamps and $ 5 Baby Bonds, postal savings accounts have become nearly extinct. What reasons can you give for this ?

126. Federal Reserve Banks.—According to a United States law of 1914 the United States was divided into twelve banking districts and a Federal Reserve Bank established at each of the following cities: Boston, New York, Philadelphia, Richmond, Atlanta, Cleveland, Chicago, Minneapolis, St. Louis, Dallas, Kansas City, and San Francisco. These banks are the bankers' banks. All national banks must and state banks may become members of the Federal Reserve System. Members are required to subscribe for stock of the Federal Reserve Bank in an amount equal to 6 % of their capital and surplus. The capital stock of banks becoming members of the system is governed by the population of the town in which the bank is located. Banks deposit their funds in the Federal Reserve Bank of their district and draw upon them just as the business man does with the ordinary bank.

127. Bank Examiners.—Banks that fail are, of course, likely not to be able to pay their depositors. In order to secure depositors against such losses, the national banks are examined from time to time by representatives from the office of the U. S. comptroller of the currency. State banks are similarly examined by state examiners. Some states, in addition to this, guarantee deposits in their state banks. What is the law in your state?

128. Bank Statements.—At regular intervals—yearly, half-yearly, or monthly—banks issue a statement regarding the financial standing such as the one below. Note that the safety of a bank depends largely upon the security of its investments. It is this fact that the bank examiners have to investigate chiefly. Poorly secured loans or very large loans made to a few investors would not be a good thing. In fact, the amount that a bank may loan to any one individual or firm is limited by law.

Condensed statement of condition of Planters State Bank at close of business, November 1, 1918:

RESOURCES

Loans and discounts	$ 730,180.60
Real estate	2,843.30
Stocks and bonds	58,943.72
Furniture and fixtures	2,240.00
Cash and sight exchange	299,059.21
	$ 1,093,266.83

LIABILITIES

Capital stock	$ 100,000.00
Surplus and profits	78,527.00
Deposits	914,739.83
	$ 1,093,266.83

EXERCISES

1. A merchant's balance Monday morning at his bank was $ 137.93. He deposited each day of the week $ 94.50, $ 97.45, $ 107.34, $ 85.46, $ 75.34, and $ 209.58 respectively. He wrote checks during the week for the following amounts: $ 38.45, $ 218.57, $ 19.65, $ 98.06, $ 145.34, and $ 78.45. What was his balance at the bank at end of the week ?

2. When Mr. Wilson's bank-book was last balanced it had a balance of $ 56.74. Since then he deposited $ 56.25, $ 17.50, $ 56.25, $ 56.25, $ 8.75, and $ 56.25. He had drawn the following checks: $ 24.57, $ 32.48, $ 89.69, $ 74.84, $ 27.19, and $ 35.26.. His check-book showed a balance of $ 13.69. Was it correct ? When his account at the bank was balanced it showed a balance of $ 88.53. Which check had not been cashed ? If Mr. Wilson had numbered his checks, he would have seen at once which check had not been cashed.

3. Give all the reasons that you can for paying bills by checks instead of by cash.

4. Find in your newspapers bank statements like the one on page 103 and bring them to school. Compare these statements with the one found upon page 103.

5. At the beginning of a week a bank has on deposit in the Federal Reserve Bank of its district $ 45,607.49. During the week it deposits $ 34,107.63 and withdraws $ 39,705.74. What is its balance with the Federal Reserve Bank at the end of the week ?

6. Why is there a need of bank examiners ? How do they help to make banking safe for the public ? Who pays the government bank examiners ? the state bank examiners ?

7. Where is the Federal Reserve Bank of your district ?

VIII

PAYMENTS AND COLLECTIONS

129. Paying Local Bills.—Except for very small amounts always pay local bills with your own private checks. Such checks should be made out to the order of the one whose bill you pay. When such a check is returned by your bank it should be filed for future possible use. Name some uses that might arise. If a check is lost no real harm results. Why?

130. Certificates of Deposit.—Upon proper payment a bank issues a certificate stating that you have deposited a certain sum to the credit of a person or a firm. Such a certificate is especially useful in making a purchase of a known amount from a person or a firm in your own city by whom you are not known.

EXERCISES

1. How should a check be written so that it may later be used as a receipt? Explain why it can be used as a receipt.

2. Write a check to Henry Doe in payment of a bill of $45.36 which you could afterward use as a receipt.

3. Suppose that the check is lost and is found by a tramp who gets it cashed. Who loses the money?

4. Suggest a specific instance in which you could use to advantage a certificate of deposit.

131. Meaning of Exchange.—It often becomes necessary to pay a debt in another city or in another country. This can be accomplished in several ways, which will be discussed in Arts. 132–138. All of these methods come under the general head of **exchange**. Unfortunately the charges for carrying out such payments are also called **exchange.**

132. Money in Letter.—For small remittances stamps may be used—1-cent stamps are preferred. Why ?

Money or other valuables may also be sent by **registered letter** for ten cents plus letter postage. If not delivered, it will be traced and if not found, refunded up to $ 50. A receipt of delivery from the person to whom the letter is sent may be obtained upon request at time of sending the letter. Unsealed mail may be insured to an amount not exceeding $ 100.
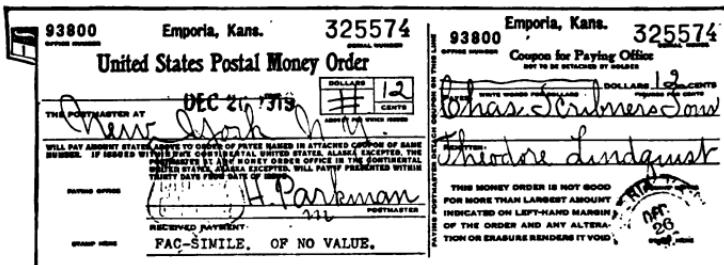
133. Postal Money Orders.—Money orders may be purchased at one post office payable at another post office. If a postal order is lost, a duplicate can be obtained from the Postal Department at Washington. The maximum amount is $ 100. For larger amounts more than one order will be required. Each order may be transferred from one person to another but once only.
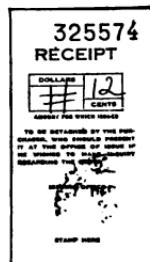
POSTAL MONEY ORDER FEES

For orders from $ 0.01 to $ 2.50 . . . . . . 3 cents.
          from $ 2.51 to $ 5.00 . . . . . . 5 cents.
          from $ 5.01 to $ 10.00 . . . . . . 8 cents.
          from $ 10.01 to $ 20.00 . . . . . .10 cents.
          from $ 20.01 to $ 30.00 . . . . . .12 cents.
          from $ 30.01 to $ 40.00 . . . . . .15 cents.
          from $ 40.01 to $ 50.00 . . . . . .18 cents.
          from $ 50.01 to $ 60.00 . . . . . .20 cents.
          from $ 60.01 to $ 75.00 . . . . . .25 cents.
          from $ 75.01 to $ 100.00 . . . . . .30 cents.

134. Express Money Orders.—Money orders similar to postal money orders are sold and cashed by express companies at their offices. Fees vary slightly from those of postal money orders. Express money orders are limited to $ 50 but may be transferred any number of times.

Money may also be sent by express directly and fully insured against loss—see· page 144.

EXERCISES

1. How can you send a letter to any person so that you may know that it is received? Suggest an instance when it may be of value to you to know that your letter has been received.

2. State the cost of sending each of the following amounts by postal money order: $ 1.35; $ 3.65; $ 8.56; $ 84.35; $ 2.50; $ 40.

3. Procure a money order application blank from the post office, and fill the same out properly for sending $ 9.45 to John Doe, 784 Fifth Avenue, New York City. How much will be the total cost?

4. Find out and compare the cost and the safety of sending the amounts stated in Ex. 2 by registered letter, postal money order, or express money order.

135. Private Checks.—There are two reasons for not using private checks for out-of-town payments. In the first place if the check accompanies an order and the sender is not known, a delay may be necessary. Why? This can be obviated by having the **check certified.** This is done by the cashier of the bank, who states upon the check that he has withheld from the deposits of the maker the amount specified to insure its payment. A private check may also be exchanged for a banker's check.

The second objection is that a fee is usually collected from the depositor of an out-of-city check for clearing-house expense. This exchange may be a certain per cent of the check, but more often is a fixed charge. In one city, for instance, the exchange is 20 ¢. What is it in your city?

86-1

THE FIRST NATIONAL BANK No.

Tulsa, Okla. *Oct 15* 191*9*

Pay to
the order of ___*Jasper Q Kapper*___ $ *225⁰⁰*

Two hundred twenty-five ＆ ⁿᵒ/₁₀₀——**Dollars**

TO SOUTHWEST NATIONAL BANK OF COMMERCE, }
18-1   KANSAS CITY, MO. }   *H. V. Sorus*
                                        PRESIDENT
                                        CASHIER

136. Bank Draft.—A **bank draft** is an order issued by your bank to you directing a bank at which it has deposits to pay a given sum to the one designated on the draft. A merchant in Jonesville buys a draft from his bank. This draft directs a New York bank to pay a stated sum to some creditor of the Jonesville merchant. The Jonesville merchant pays his bank the face of the draft plus the exchange.

Sometimes the exchange is a per cent of the face of the draft, but more often it is a fixed charge. What is the charge in your city? No exchange is paid by the one to whom a draft is payable, and he is also assured that it will not be rejected, as may be the case with a private check. Many buy drafts made out to themselves and then indorse them to whomever they have remittances to make. In indorsing a draft always make it payable to **the order** of the person or firm to whom it is sent. Why?

EXERCISES

1. State a case in which you could use a certified check to advantage.

2. Write a draft as if made out by a bank in your city, ordering a New York bank to pay you $450.

3. Indorse this draft, making it payable to the order of Richard Roe, Albany, N. Y.

4. What will be the exchange if the rate is $\frac{1}{10}$ %?

5. How would you send a remittance for 7 lb. butter at 35 ¢ per pound to Mr. White in the country?

6. Mrs. East made a trip of m miles. What was her total fare at $2\frac{1}{2}$ ¢ per mile with $ d for Pullman ticket?

7. If Mrs. East went 2680 mi. and the Pullman ticket was $14.40, what was her total fare? Mrs. East paid for the tickets with a certified check. Why?

8. Nina has N $50 registered $4\frac{1}{4}$ % government bonds. How large is her semi-annual interest check?

9. A house is L ft. long, W ft. wide, and H ft. high. The window surface is estimated to equal that of the gables and the porches. What is the cost of the paint needed at $ D per gallon, if each gallon covers 250 sq. ft. of surface?

10. What is the cost of the paint for a house $44\frac{1}{2}$ ft. long, 25 ft. wide, and 20 ft. high at $2.47 per gallon? Write a check in payment for the same.

137. Foreign Exchange.—In making quotations for remitting money to foreign countries, banks state the price in United States money of each foreign denomination. The following gives a few of the most important quotations.

TABLE OF FOREIGN EXCHANGE

Currency	On	Parity	Cable Rates Aug. 8, 1919	Purchasing Power of a Dollar Abroad Per Cent	Purchasing Power of a Foreign Unit Per Cent
Pounds	London	4.8665¾	4.33½	112.3	89.0
*Francs	Paris	5.1825	7.75	149.5	66.9
*Francs	Switzerland	5.1825	5.60	108.1	92.5
Florins	Amsterdam	.4020	.37¾	107.6	92.9
*Lire	Italy	5.1825	8.90	171.7	58.3
*Drachmai	Greece	5.1825	5.18	99.9	100.1
Kroner	Copenhagen	.2680	.2200	121.8	82.1
Kroner	Sweden	.2680	.2490	107.7	92.9
Kroner	Norway	.2680	.2380	112.6	88.8
Pesos	Argentina	.4245	.4230	100.4	99.6
Pesetas	Spain	.1929	.1905	101.2	93.8
Yen	Yokohama	.4985	.5125	97.3	102.8
Marks	Berlin	.2382	.06125	389.0	25.7

* Quotation shows number of foreign units which can be obtained for one United States dollar.   All other quotations show value of one foreign unit in United States dollar.

To make a remittance to London of £ 350 by draft will cost 350 × $ 4.33½.   How much is this?

EXERCISES

1. What will be the cost of a draft upon Argentina for 2000 pesos? for 1650 pesos?

2. What will be the cost of a draft upon Yokohama for 8450 yen? for 2825 yen?

3. What is the cost, C, of a P pound draft on London? Solve the equation for P.

4. What will be the cost in London of a draft upon New York for $ 650? for $ 1540?

5. What will be the cost in Paris of a draft for $ 200?

6. What will be the cost of a draft upon Genoa, Italy, for 3450 lire? for 1560 lire?

7. Explain how the last two columns in the table on page 110 were found.   State this by an equation.

8. Make use of your equation to verify two of the per cents found in each column.

9. What will be the cost of a draft to Sweden to pay a bill of 7605.80 kroner for fine cutlery ?

138. Sending Money by Telegraph.—It is sometimes necessary to send money as quickly as possible. This is accomplished through the telegraph companies. The telegraph office from which the money is forwarded sends a telegram directing its office in the city where the money is needed to pay a stipulated amount to a designated person ·or firm upon proper identification. A charge is made of a 15-word message in addition to 25 ¢ for amounts not over $ 25, 85 ¢ for amounts between $ 25 and $ 100, and 25 ¢ per $ 100 or fraction thereof above $ 100. This is the most rapid as well as the most expensive means of sending money.

Telegrams are also used in directing banks or others to pay certain sums to persons or firms under varying conditions. Such messages are generally sent in code. Why ?

EXERCISES

1. The charge of a 15-word telegram from A to B is 75 ¢. What is the total amount paid in sending $ 350 from A to B by telegraph ?

2. Discuss fully the principal advantages and disadvantages of each form of exchange found in Arts. 132–138.

3. State some occasions on which you would use each in preference to any of the others.

4. What would be the cost of sending $ 240 by each of the methods discussed ? Make the exchange of drafts $\frac{1}{10}$ % and the cost of a 15-word telegram to the place to which the money is to be sent 50 ¢.

5. Write a telegram directing your bank in some other city to pay to a certain person $ 150 from your account.

139. Commercial Payments.—Very little money actually changes hands in the commercial transactions in any city. Mr. A. pays a bill to Mr. B. by having his bank-account debited the amount of the bill and having Mr. B.'s bank-account credited a like amount. How is this done by a check? It is an exchange of credits; not an exchange of money.

Similar transactions take place between various sections of the country and also between different countries. For instance, in the early fall a great deal of grain is shipped from the West to the East. For payment the East does not send money to the West but gives the West credit in the Eastern banks, generally in New York banks, for the value of the grain. The West owns the money, but it is in Eastern banks. A grain dealer in Jonesville sells wheat for $ 5000 to New York. His bank in Jonesville credits him with $ 5000 which he can check against at any time. The Jonesville bank has some particular bank in New York with which it does its Eastern business and in which it always keeps some deposit. The firm buying the wheat now deposits $ 5000 to the credit of the Jonesville bank in the New York bank at which the Jonesville bank keeps its deposits. Note that the firm buying the grain has paid out $ 5000; the grain dealer has received $ 5000 to his credit at the Jonesville bank; the Jonesville bank has paid out $ 5000 from its funds at Jonesville but has been given an equal amount of credit with its New York bank. Explain this transaction fully.

Later when the merchants of Jonesville buy goods from the East they do not send money. The Jonesville merchants buy from the Jonesville bank a part of its credit in the New York bank in the form of a draft. This credit the Jones-

ville merchants then forward to their creditors in the East, who deposit the same in their banks. Note that the Jonesville merchants have paid for their goods; the Jonesville bank has received money or credit in Jonesville but has turned over a like amount of its deposits with its New York bank; those from whom the goods were purchased have been paid. At neither time was any money sent from one section of the country to the other.

In case the credit at either part of the country becomes very heavy it may be necessary to send some money to "balance the trade," as it is called. If the Jonesville bank has very much credit in New York, it may be glad to sell some of this credit below its actual value. Exchange on New York is then **below par.** At that particular time exchange in the East on the West will be **above par** or **at a premium.** Explain. The exchange may also be reversed. Explain when that would happen.

The English pound is valued at $4.866 but during the war and immediately following, exchange on London dropped to considerably below $4 for a pound. What reasons can you give for this? What is exchange on London now? What reasons can you give for the variation?

EXERCISES

1. Explain the procedure of paying a debt in your own city with a check.

2. Explain fully the procedure of paying a debt in another town by an exchange of credits.

3. Find the cost of a New York bill (called draft) of $2560 with exchange 1 % premium; $\frac{1}{2}$ % below par.

4. What is the cost of a draft of $ D when exchange is E %?

140. Travellers' Checks.—Banks and express companies issue a form of checks for travellers. These are like the above and put up as below. Checks are for $ 10, $ 20, $ 50, $ 100, and $ 200. The usual fee is $\frac{1}{2}$ % of the total face value of the check sold. Each check bears the signature of the official selling the checks and of the purchaser. In order for the traveller, who is likely not known, to get a check cashed he merely signs the check again in the presence of the one paying it. The duplication of his old signature on the check is his identification as the owner of the check. If the checks are lost or stolen, immediately notify the office that sold them or agents authorized for that purpose to stop payment.

141. Letters of Credit.—Most banks also issue **letters of credit** for travellers in foreign lands. These contain the signature of the owner of the letter, signature of the official selling it, and names of the institutions from which funds may be obtained. The traveller presents his letter at any one of the institutions named therein and

signs a receipt, his name corresponding to the signature upon the letter, for any amount paid to him. The amount paid and the name of the institution paying it are then recorded upon the letter. The sum of the amounts paid must, of course, not go beyond that for which the letter was bought. The usual fee is 1 % of the amount stated in the letter.

Travellers' checks are also issued for use in foreign countries. Many prefer these to the letter of credit for greater convenience in daily use.

EXERCISES

1. Make out a sample of a traveller's check in favor of yourself for $ 20 and sign it John Doe, cashier of the Emergency Bank.

2. State the denominations in which you would desire to have traveller's checks for $ 100; $ 200; $ 500. What is the cost of each ?

3. Use the rates of exchange found on page 110 and find how much should be paid for a letter of credit so that the following amounts may be obtained: £ 50 in London, 1000 francs in Paris, and 800 lire in Rome.

4. Estimate your expense for a month's trip during your vacation. How would you carry the money necessary ? Plan such a trip for your class as a whole.

5. Estimate the expense of a trip to some foreign country you have studied about. Explain how you would carry the necessary funds.

6. J. M. Weeks bought of the White Pine Lumber Co., Vancouver, Canada, 45,600 ft. lumber at $ 45 per M, 23,400 at $ 38, and 68,000 ft. at $ 39.50. How much would he remit if exchange on Canada was .925 ?

7. How much would he remit at the present exchange rate ?

PRODUCE NATIONAL BANK   83-75

Wichita, Kan. Dec 17 1919

At sight   Pay to the

order of Garden City State Bank $ 345 50/100

Three hundred forty-five & 50/100 Dollars

Value received and charge the same to account of

To Henry Post   } Thomas Macklin Co.

Garden City, Kan.

142. Sight Drafts.—Banks also act in the capacity of collecting agencies. Suppose that Henry Post owes Thomas Macklin Co. a bill for $ 345.50. The Thomas Macklin Co. can make out a **sight draft**, like the above, that they present to their bank, which forwards it to its bank representative in the city where Henry Post carries on his business. This latter bank presents the sight draft to Henry Post for payment and gives him a receipt if he pays the same. The money is then returned to the Thomas Macklin Co. through its bank, less a small charge for collection. If Henry Post does not "honor"—does not pay—the draft, it is returned with that statement. This is usually one of the first steps taken to secure payment on an out-of-town bill. If Henry Post **dishonors** too many bills, his credit soon suffers. He becomes known as one who does not meet his payments promptly and finds it hard to secure credit in buying goods for his business.

143. Time Drafts.—On the opposite page will be found a **time draft.** This is just like the sight draft except that it will not be due for 60 da. This draft is handed to a bank just as was the sight draft and by it forwarded to its representative in the city where Joseph M. Doughley does business. This latter bank presents the draft to Joseph M.

Doughley, who writes across it the word **"acceptance"** and signs his name or initials, if he acknowledges the correctness of his indebtedness to the Horning Manufacturing Co. Otherwise he rejects it. If the draft is "accepted," it then serves as a contract for the Horning Manufacturing Co., and they further can leave it with their bank as security in borrowing money.

EXERCISES

1. William Wasman owes $ 235.25 to John Crews Co., for which he has made no remittance. Write a sight draft for the bill to be turned over to the bank of John Crews Co.

2. Jacob, Stuckart & Co. have sold a bill of goods to Z. U. Mists for $ 2560, which will be due in two months. Write a time draft which Jacob, Stuckart & Co. may get accepted and afterward use as collateral in obtaining a loan from their bank.

3. Suggest and write another sight draft.

4. Suggest and write another time draft.

5. The Scroggin Manufacturing Co. holds accepted drafts to the value of $ 17,500, which they leave with their bank as security for a 60-da. loan of $ 15,000. What will be the net proceeds if the discount rate is 8 %?

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IX

INSURANCE

144. Principle of Insurance.—As a rule the losses in any community due to tornado, hail, fire, or other accidents fall upon a very few, but are exceedingly heavy upon these few. This loss also comes all at once. The same is true of losses to families through sickness, accident, or death of the chief wage-earner. Through the medium of insurance companies these losses are distributed among many of the people of a community and for each person they are distributed over a long space of time.

Suppose that all of the property owners in a community banded themselves together to share each other's losses by fire. Suppose, further, that the total valuation of the property of these men is $ 800,000 and that one of them sustains a fire loss of $ 1500. The question then arises, how much should a man whose property is valued at $ 3000 pay into the helping fund ? The $ 1500 is .002 or .2 % of the total property owned by these men. How do you find this? Hence, the man owning the property worth $ 3000 pays into the fund .2 % of $ 3000. How much is this ? Would it be better business for the man owning the $ 3000 property to save the money he thus paid to the general fund and to *have carried the* risk of losing $ 3000 all by himself ?

145. Mutual Insurance Companies.—Groups of men banded together as mentioned on the last page are called **mutual insurance companies.** Each member pays a little more than the amount of losses to defray necessary expenses. The duties of the manager of such a company are to look over properties upon which insurance is desired, to decide the amount of loss sustained by any member through a fire, to notify the members of payments to be made to cover losses, and any other duties that may be needed. In addition to mutual fire insurance companies there are also many mutual life insurance companies, including the fraternal orders.

146. Private Insurance Companies.—Most of the insurance is carried on by private companies, however. They sell insurance at a definite fixed price, just as any other commodity is sold. In the place of paying only when losses occur and then a varying amount, those who insure in private companies pay a certain fixed amount when the insurance is taken out. The cost of such insurance for every $ 100 of property insured and the method of fixing this cost will be discussed later. (See Art. 149.)

147. Expressions Used.—The written agreement between the company and the person whose property is insured is called a **policy.** In the policy is stated the value at which the property is insured, called **face of the policy;** the **rate** per $ 100 of insurance; total amount paid for the insurance, called **premium;** time of duration of the policy, called **term.** An insurance agent or company is often spoken of as an **underwriter.**

148. Classes of Insurance.—We shall here study only the two most common classes; namely, fire and life insurance. Some of the more special forms are marine, live stock, crop, plate glass, burglary, accident, sick benefit.

149. Rates.—Rates of private insurance companies differ greatly, depending upon the risk of the property insured. For instance, the rate will be higher on a frame building than on a stone or brick building used for the same purpose and situated similarly. Why? A dwelling-house in a residence neighborhood has a lower rate than a factory in a manufacturing district. Why? The rates charged are determined after a careful study of fire losses that have occurred in each form of risk during a long period of time. Rates also may vary as to cities according to the fire protection provided. Fire rates in the United States are higher than in most other countries because our fire losses are so high. The total fire losses in the United States in 1915 were $ 172,033,200. This is about $ 1.71 per each person, called **per capita.** How is that found? In Europe the loss was only 33 cents per capita for the same time.

Dwellings in a city are generally given the same rate, with the premiums $2\frac{1}{2}$ times the yearly premium for 3 yr. and 4 times the yearly premium for 5 yr. Each business building and factory usually has its own rating. Why? As stocks of goods vary greatly they are, as a rule, insured for only a year or less.

The following are about the rates of insurance per $ 100 in a Middle Western city of 10,000 inhabitants possessing good fire protection: dwelling-houses and contents—brick, 25 ¢, and frame, 40 ¢; brick business buildings, 75 ¢ to $ 3; dry-goods stock, $ 1.10 to $ 1.60; barn and live stock, $ 3 to $ 4; hay and grain, $ 3.35 to $ 4.25; mills and factories, $ 3.75 to $ 5. Why these wide differences?

EXERCISES

1. Who needs most to carry fire insurance, a man who *possesses* considerable property or one with limited means?

2. The total property listed in a mutual fire insurance company is $ 1,340,000. One of the members sustains a loss of $ 1600. What will be the assessment of a man who carried $ 4500 of insurance in the company ?

3. Basing the rates upon the per capita fire losses as given in Art. 149, how many times the European rates should be those in the United States ?

4. The rate of insurance of a certain business house in a city in the United States is $ 1.85 per $ 100. How much would this be at the European rates ?

5. A business building is insured at $ 12,500, the rate being $ 1.93. Find the premium.

For dwelling-houses use rates of Art. 149.

6. Find the premium on a frame dwelling and contents insured at $ 3500 for 1 yr.; for 3 yr.; for 5 yr.

7. Is it more economical to insure a house yearly or to pay the premium for 3 yr. at one time ?

8. Make and solve a problem which will show the point raised in Ex. 7.

9. Find the yearly premium for a factory insured for $ 45,000 at the rate of $ 4.65.

10. Suppose that the factory in the last problem is equipped with fire-extinguishers costing $ 1400, thereby bringing its insurance rate down to $ 3.15. Find the saving in insurance per year.

11. How long would it take the above factory to save the cost of the fire-extinguishers ?

12. Use literal numbers instead of those in Ex. 2 and state the answer to the question by an equation.

13. Solve the equation formed in Ex. 12 for each of the literal numbers it contains.

TABLE GIVING PER CENT OF ANNUAL PREMIUM FOR DAYS

1 day........ 2 %	18 day.......16 %	105 day......45 %
2 " .... 4 %	19 " ....16 %	120 " ......50 %
3 " .... 5 %	20 " ....17 %	135 " ......55 %
4 " .... 6 %	25 " ....19 %	150 " ......60 %
5 " .... 7 %	30 " ....20 %	165 " ......65 %
6 " .... 8 %	35 " ....23 %	180 " ......70 %
7 " .... 9 %	40 " ....26 %	195 " ......73 %
8 " .... 9 %	45 " ....27 %	210 " ......75 %
9 " ....10 %	50 " ....28 %	225 " ......78 %
10 " ....10 %	55 " ....29 %	240 " ......80 %
11 " ....11 %	60 " ....30 %	255 " ......83 %
12 " ....12 %	65 " ....33 %	270 " ......85 %
13 " ....13 %	70 " ....36 %	285 " ......88 %
14 " ....13 %	75 " ....37 %	300 " ......90 %
15 " ....14 %	80 " ....38 %	315 " ......93 %
16 " ....14 %	85 " ....39 %	330 " ......95 %
17 " ....15 %	90 " ....40 %	360 " ......100 %

150. Short Terms.—The above table gives the per cent of the various yearly rates that are charged for short-term insurances. This is the standard short-term schedule. For instance, the table says that the insurance for 20 days is to be 17 % of whatever may be the yearly premium.

151. Refunds in Cancelling Policies.—If a house is sold and the buyer does not care to continue the insurance, it may be cancelled and a certain per cent of the premium paid returned to the policy-holder. Again, goods may also be shipped into a state in which the particular insurance company does not do business. In cancelling a policy thus the insurance company charges insurance at the short-term rate and refunds the difference between this and the yearly premium paid. Thus, if a policy is cancelled after it has been in force 20 days, only 17 % of the yearly premium will be retained. What per cent will be returned to the holder of a one-year policy ? It also happens sometimes that the in-

surance company wishes to cancel a certain risk. In that case the company may retain only that portion of the yearly premium that the time it has been in force is of a year. In the above case the company could retain only $\frac{20}{360}$, or $\frac{1}{18}$. What per cent is this?

<div align="center">EXERCISES</div>

1. A stock of goods valued at $ 8500 is insured for 30 days at a yearly rate of $ 1.85. Find the premium.

2. A stock of goods is insured for $ 16,500 at $ 2.15. Find the premium for one year. What is the refund if cancelled after it has been in force 9 months?

3. Find the yearly premium on a frame dwelling insured at $ 2500; see Art. 149. The company decides to cancel the policy after it has been in force 8 months. How much refund must the company make?

4. Find the insurance for 10 days on a stock of grain insured at $ 850 if the rate is $ 2.85.

5. If a stock of goods is insured monthly, how many times the yearly rate will this be after 12 months?

6. A stock of goods is insured for $ 12,500 at a yearly rate of $ 2.65. Find the monthly premium.

7. Mr. Baker insured his household goods for $ 900 at $ 1.35 per hundred, 2 mo. after moving into a city. How much did he save by waiting? A loss by fire of $ 280 during this time would have been how many times the premium saved?

8. How much would Mr. Tight save yearly by not insuring his house for $ 4000 at a rate of 30 ¢? A loss by fire of $ 1400 is how many times the premium? Can he afford to take this risk?

9. Have you seen an insurance policy? Can you obtain one that is not in force to show to the class?

152. Valued Policy.—A **valued policy** is one in which the loss to be paid is a definite amount. It is used in the cases where the loss would be total—a ship, a horse, or a cow.

153. Open Policy.—In the **open policy** the insurer agrees to pay the damages incurred up to a certain stipulated amount. It is used in cases where the loss is seldom total; the cargo of a ship and a herd of cattle. Fire insurance policies are open policies. Where losses occur that are covered by an open policy an appraisement of the amount to be paid by the insurance company is made by an insurance-adjuster. As loss by fire is very seldom total, it is a good plan to make the face of such a policy about three-fourths actual value. Why ?.

A few states have laws to the effect that only 80 % of the value of the property can be claimed in case of loss.

Large risks are sometimes divided among several companies. Where one company writes a very large policy it often does not desire to carry all of the risk and reinsures in other companies. Whenever losses occur in such cases they are divided proportionately among the companies that have written the policies.

EXERCISES

1. How large a policy would you take on a dwelling valued at $ 4000 ? at $ 6000 ? at $ 3500 ? at $ 2500 ?

2. Find the premiums for 3 years and for 5 years that should be paid upon frame dwellings valued at $ 2000; valued at $ 3000.

3. A factory and its contents are valued at $ 85,000. How much insurance ought to be carried ? What would be the yearly premium, if the yearly rate is $ 3.85 ?

4. A building valued at $ 35,000 is insured for $ 30,000. If the building is totally destroyed by fire, what will be paid in one of the states in which only 80 % of the total valuation is paid ? What will be paid in one of the other states ?

5. A stock of goods valued at $ 18,500 is insured for three-fourths valuation for a period of 30 days. Find the premium if the yearly rate is $ 2.45.

6. Mr. Williams insured his brick house in one company and household goods in another, each at three-fourths valuation. The house is valued at $ 5000 and the household goods at $ 1500. Find the premiums paid each company for a term of 3 years; for a term of 5 years. (See Art. 149.)

7. After a fire in Mr. Williams's home it was found that the house was damaged $ 3300 and the household goods $ 650. How much did each company pay him ?

8. A wholesale company contemplating the erection of a new building submits the plans to an insurance-adjuster who suggests certain changes and additions whereby their insurance rate can be cut from $ 1.95 to $ 1.05. The first contemplated cost was to be $ 34,600 and the additional changes would cost an extra $ 3600. If insured for three-fourths of valuation, what would be the yearly saving in premium ? How long would it take to save the extra outlay ?

9. A factory is insured in three insurance companies for $ 25,000, $ 20,000, and $ 15,000 respectively. What part of a loss of $ 1200 must each pay ?

10. An insurance company which wrote a policy for $ 200,000 decides to reinsure in two other companies for $ 50,000 each. What per cent of any loss should each company pay ?

154. Life Insurance.—There are many forms of life insurance policies. They are uniform in one or two particulars. The company agrees to pay the total face of the policy at the death of the insured, providing the premiums have all been paid up to that time. This payment is made to the person or persons, called **beneficiaries,** stated in the policy. Some policies require the payment to be made to the estate of the deceased.

155. Whole Life.—This policy calls for certain premiums to be paid at regular intervals of a year, 6 months, or 3 months, during all the time that the policy is in force.

156. Limited Payments.—Premiums are paid at regular intervals, but for a limited number of years—15 yr., 20 yr., or more. A common form, generally known as the 20-payment life, has 20 yearly payments. After the required number of payments have been made the insured has a **"paid-up policy."** The insurance remains in force but no further payments are required. Such policies usually contain a clause making it possible for the insured to surrender the policy after three yearly premiums have been paid. The more premiums that have been paid the more the insurance company will pay for the surrender of a policy. Why are the rates higher than in the whole-life policies?

157. Endowment.—Endowment policies have limited payments like those described in Art. 156 except that the policy may be cancelled at the end of the term for its face value. The premiums are considerably higher. Why?

Any of the above policies may be written so that the insured participates in the gains of the company. Payments of these gains, called **dividends,** may be called for yearly or permitted to accumulate.

AGE	WHOLE LIFE	10-YR. PAYM'T	20-YR. PAYM'T	15-YR. ENDOW.	20-YR. ENDOW.	5-YR. TERM	AGE
20	$ 18.01	$ 45.51	$ 27.78	$ 65.14	$ 47.54	$ 10.43	20
24	19.68	48.35	29.59	65.48	47.91	10.73	24
27	21.15	50.74	31.12	65.79	48.27	11.02	27
30	22.85	53.38	32.83	66.18	48.71	11.37	30
33	24.84	56.31	34.75	66.66	49.28	11.82	33
36	27.18	59.55	36.93	67.27	50.01	12.42	36
38	28.95	61.91	38.56	67.77	50.63	12.93	38
40	30.94	64.44	40.24	68.38	51.39	13.55	40
45	37.09	71.66	45.69	70.58	54.15	15.88	45
50	45.45	80.51	52.83	74.30	58.76	20.35	50
55	56.93	91.42	62.66	80.45	66.32	28.17	55

Yearly premiums for each $1000 of insurance.

158. Term.—A 5-yr. term policy calls for 5 yearly premiums and goes out of force at the end of 5 yr. If the insured desires to continue the insurance, another policy must be taken out at the higher rate for a person 5 yr. older and another physical examination must be passed. Term policies are also written for 10 yr., 15 yr., and 20 yr.

159. Accident and Sick-Benefit Insurance.—The insured pays certain premiums as in life insurance and receives stated sums in case of accident or sickness.

EXERCISES

1. What is the yearly premium on a whole-life policy of $ 2000 at age 36 ? of $ 5000 at age 24 ?

2. What is the yearly premium on a 20-payment life policy of $ 3000 at age of 20 ? of $ 5000 at age of 33 ?

3. Suppose dividends for the policies in Ex. 2 are 15 % of premiums, to what could each premium be reduced ?

4. Why do rates advance with age ? Why do rates differ in the various forms for any age, as 30 ?

5. The yearly dividends on a 20-payment life policy average $ 4.37 for one insured at 27 yr. What per cent of the premium is the dividend ?

6. A person insured at 20 yr. can secure a paid-up 20-payment policy after making 16 payments by permitting dividends to accumulate. How much has then been paid ?

7. Suppose that $ 27.78 was deposited the first of January every year for 16 yr. at a savings-bank paying 4 % interest compounded annually. How much would the depositor have to his credit at the end of the 16 yr., providing no withdrawals were made ?   (See page 86.)

8. Why is the amount found in the last exercise the true cost of the insurance mentioned in Ex. 6 ?

9. Suppose that the company will cancel the policy and pay the one insured $ 500 per $ 1000 at the end of 16 yr. How much less is this than the cost of the policy to the one insured ? What does the insured receive for the extra cost ?

10. To the same axes make graphs of the premiums for whole-life and some other form of policy on page 127. What do the graphs show ? A more complete table can be secured from an insurance agent if thought best.

· **160. Bonding Companies.**—Many who accept positions of trust are required to furnish a bond as to their honesty and faithfulness. Employees handling money, as the cashier of some firm, come under this rule. The bond states for how much the signers of it are responsible for any losses incurred by the employee bonded. Years ago it was customary for one required to furnish a bond to get it signed by his relatives or friends. Bonding companies now sign these bonds insuring an employer against loss by an employee just as a fire-insurance company insures an owner of a building against loss by fire.

161. Risks Accepted.—Not all who apply for a bond from a bonding company can secure one. The record of each applicant is looked into fully. Some of the questions asked of teachers and former employers are: if they have known the applicant to *speculate, gamble, bet, be extravagant, keep bad associates, smoke cigarettes.*

162. Rates.—The rates paid to bonding companies vary greatly, but are a certain per cent of the amount named in the bond. They range from $\frac{1}{4}$ % to $1\frac{1}{2}$ % with an average of $\frac{1}{2}$ % per annum.

EXERCISES

1. Why is it more businesslike to pay for a bond from a bonding company than it is to secure the same service from your friends?

2. Mr. Jones accepted a position that required a bond of $ 5000, which he permitted his friends to sign for him. He afterward loaned one of these friends $ 85 that he could not collect. What per cent of premium did he really pay for his bond?

3. Does a bonding company have any right to ask the above private questions regarding one who desires a bond from them? Why are these perfectly good business questions for the bonding company to ask?

4. A bank cashier has himself bonded for $ 25,000. What premium does he pay the bonding company at $\frac{1}{4}$ %?

5. A firm which contracts to build a factory has to furnish a bond for $ 150,000. What will be the premium at $1\frac{1}{4}$ %?

6. Because a young man had been betting he could not secure a bond, and was thus forced to work for $ 85 in the place of $ 125 per month. How much salary did he lose one year because he had been betting?

X

TAXES

163. Public Funds.—Who pays your teacher? the sheriff in your county? the governor of your state? the President of the United States? All funds collected to pay public officials and for carrying out public improvements are called **taxes.** What public improvements have recently been carried out in your community? Who paid the taxes that provided the necessary fund?

164. State and Local Taxes.—Taxes collected for carrying on the public work in the state, county, city, and school district will be discussed in Arts. 165–172.

165. Poll Tax.—Poll taxes are assessed against male voters in the country and smaller cities to keep up the roads. In some places it is called road tax. The **automobile tax** of states and the **wheel tax**—a tax paid upon any vehicle—of cities are forms of road tax.

166. Personal-Property Tax.—An official, called **assessor,** who is elected by the people, calls upon each owner of personal property in his district yearly—usually in the spring—for a schedule of the personal property owned upon a certain day. In a few months thereafter taxes on the property become due. If not paid within a certain time, these taxes are collected by the sheriff.

When are taxes due in your state?

167. Real-Estate Taxes.—Real-estate taxes are assessed against the property. The taxes are due at the time of the personal-property taxes and if not paid within a certain length of time a certain per cent is added monthly for a few months. If not paid by this time, the property is sold for taxes. The valuation of real-estate properties is made by real-estate appraisers. A county or city board of equalization meets yearly, after the real-estate appraisement has been made, to hear and to adjust complaints of owners.

168. Rate of Taxation.—The rate of tax upon assessed valuation is variously given as per cent, mills per dollar, or cents per $ 100. Properties are also assessed variously in different places, from full valuation to as small a part as only $\frac{1}{3}$ of actual valuation.

EXERCISES

1. Name as many uses as you can of taxes by the state; by the county; by the city; by the school district.

2. Are poll taxes collected where you live ? How much are they ?

3. Name at least ten articles that come under personal property. In your state you schedule the property that is owned on what date ?

4. In a certain state the tax to support state institutions is 1.2 mills per dollar. How much is this for a man who owns property valued at $ 4500 ? at $ 18,400 ?

5. The tax rate in a city is 3.8 %, where $\frac{1}{3}$ of actual value is assessed. What is the tax on $ 6500 actual valuation ? on $ 34,500 actual valuation ?

6. Arrange in order the following rates: 4.7 mills on the dollar; 46.5 ¢ per $ 100; .48 %.

7. A man living in a certain state pays 4.3 % tax on ⅓ valuation, while a man in another state pays 1.5 % on total valuation. Which pays the higher rate ?

8. Which is the higher rate, 8.5 mills per dollar on actual valuation or 3.7 % on ¼ valuation ?

The following gives the tax rates in cents per dollar on total valuation for a certain city in the year 1917.

General...... .1546	Spec. Improv. .15	Total city..... .64
Interest...... .18	Sewer....... .0034	School........ .70
Sinking...... .04	Band........ .02	County....... .445
Park........ .012	Light........ .04	State......... .145
Library...... .04	Total city.... .64	Total........1.93

9. Explain the meaning of each item in the above table.

10. What part goes to the state ? to the county ? to the city ? State each as a per cent.

11. What will be the total tax on an assessed valuation of $ 2350 ? of $ 850 ? of $ 56,300 ?

12. Henry's and Mamie's father is assessed on $ 9500. How much does he pay this year for their education at the above rate ?

13. How much tax will a man pay on $ 650 of personal property and $ 5600 of real estate ? How much goes to the state ? to the county ? to the city ?

14. State as an equation that the tax, *t*, equals the value assessed, *v*, times the tax rate, *r*. Solve the equation for *r*; for *v*.

15. Use literal numbers and answer each question in Ex. 10 by an equation.

16. Find the part of the total tax that goes to each if *the city gets .59*, schools .63, county .39, state .125.

THE TABLE FOR RATE 1.97 %

	0	1	2	3	4	5	6	7	8	9
0		.0197	.0394	.0591	.0788	.0985	.1182	.1379	.1576	.1773
1	.197	.2167	.2364	.2561	.2758	.2955	.3152	.3349	.3546	.3743
2	.394	.4137	.4334	.4531	.4728	.4925	.5122	.5319	.5516	.5713
3	.591	.6107	.6304	.6501	.6698	.6895	.7092	.7289	.7486	.7683
4	.788	.8077	.8274	.8471	.8668	.8865	.9062	.9259	.9456	.9653
5	.985	1.0047	1.0244	1.0441	1.0638	1.0835	1.1032	1.1229	1.1426	1.1623
6	1.182	1.2017	1.2214	1.2411	1.2608	1.2805	1.3002	1.3199	1.3396	1.3593
7	1.379	1.3987	1.4184	1.4381	1.4578	1.4775	1.4972	1.5169	1.5366	1.5563
8	1.576	1.5957	1.6154	1.6351	1.6548	1.6745	1.6942	1.7139	1.7336	1.7533
9	1.773	1.7927	1.8124	1.8321	1.8518	1.8715	1.8912	1.9209	1.9406	1.9603

169. Use of Tables.—Tables like the above save time in computations. The first digit of a number is in the column to the left and the second digit is in the row at the top. To find the tax on $ 45, begin with 4 at the left and follow that line to the column with 5 at the top, where the tax, $ 0.8865 or $ 0.89, is found. How many times this is the tax for $ 450 ? $ 4500 ? $ 45,000 ? The tax for $ 6350 is the tax for $ 6300 and for $ 50, or $ 124.11 + $ 0.99 = ?

EXERCISES

From the above table find the tax for the following:

1. $ 450   **4.** $ 4500   **7.** $ 2400   **10.** $ 50,600

2. $ 860   **5.** $ 6050   **8.** $ 3475   **11.** $ 34,000

3. $ 6000   **6.** $ 8300   **9.** $ 2385   **12.** $ 18,200

13. Is it more economical to borrow money at 8 % for 3 mo. to pay a tax of $ 134.28 or to accept a penalty of 1 % extra tax each month and $ 1.36 other extra expenses ?

14. John pays $ 2 tax on his dog, Yankee. That is the interest on how much money at 5 % ?

15. What is the dog tax in your city or county ? This is the interest on how much money at 5 % ?

170. Spreading the Tax.—Suppose that the assessed valuation of a city and the tax necessary to pay its expenses for the coming year are both known, how can the tax rate be found? Finding the tax rate is called **"spreading the tax."**

EXERCISES

1. Explain the tax equation $T = rV$. Solve for r.

2. The assessed valuation of the property in a county is $72,000,000. Read this number. If $283,500 will be necessary to run the county government for one year, what will be the tax rate?

3. State the tax rate in the forms given in Art. 168.

4. The assessed valuation of the county for which the tax rate is given on page 132 is $46\frac{1}{2}$ million dollars. Express this in numerals. Find from the table the amount of the state tax paid by that county.

5. How much did the above county require to run its government during the year?

6. The city mentioned in the tax table on page 132 has an assessed valuation of $12,750,000. What did the city pay in school taxes?

7. A small city with an assessed valuation of $1,290,000 will need $8300 during the year for its schools. What will be its school tax rate?

171. Special Assessments.—Improvements which benefit only a limited district, as paving streets, laying water-mains, constructing sewers, and so on, are assessed against the property benefited. These are paid in yearly instalments for a period of 5 yr., 10 yr., or more.

Do you know of any such assessment? Make up a problem about one.

172. Licenses.—Certain businesses pay a fee to the city in which they are carried on. This fee is called a **license.**

EXERCISES

1. A grocer pays a tax of 1.6 % on full valuation of $ 1850 property and $ 18 license. What is his total tax ?

2. A merchant paid a tax of 4.3 % on ⅓ valuation of $ 5600 property, a poll tax of $ 3, a license of $ 15 to operate a market, and $ 2 wheel tax for each of three vehicles. What was his total tax ?

3. Mr. Georgias bought a lot for $ 1250. He paid a yearly tax at the rate of $ 1.38 per $ 100 and a special assessment of $ 35. What will be his total taxes ?

4. If•Mr. Georgias could have loaned the money at 6 % which he invested, what would the lot cost him at the end of the first year ?

5. What will the $ 1250 amount to in 3 yr. at 6 % compound interest ?

6. What will his taxes and assessments amount to in 3 yr. at compound interest ?

7. From Exs. 5 and 6 what does the lot cost him at the end of 3 yr. ?

173. National Taxes.—In Arts. 174–179 will be discussed taxes paid to carry on the national government. In 1915 the national expenditure was $ 776,544,125, while for 1918 it was estimated to be $ 18,775,919,000. Why this great difference ? How much is this difference ? Name as many expenses as you can of the national government.

174. Internal Revenues.—The federal government collects a tax upon the manufacture of several products, as spirits and tobacco. It also requires a tax upon the sale of many articles, as perfumes, patent medicines, playing-cards, some drugs, and so on.

175. Income Tax.—Since 1913 the federal government has taxed incomes. Only net incomes are taxed; those that remain after all expenses necessary in producing the income have been deducted. Living expenses are not thus deducted. A merchant sells goods during the year for $ 76,500 which cost him $ 58,370 while the expenses of conducting his business are $ 12,940. His net income is

$$\$ 76,500 - (\$ 58,370 + \$ 12,940) = ?$$

Write an equation expressing the general net income.

A single person has an exemption of $ 1000, and the head of a family $ 2000 with $ 200 for each child. The tax is 4 % on the first $ 4000 above exemption and 8 % thereafter. An additional tax, **surtax,** is also paid on incomes above $ 5000 as shown in the partial table below.

1 % $ 5,000 to $ 6,000	. . . .	
2 % 6,000 to 8,000	52 % $ 100,000 to $ 150,000	
3 % 8,000 to 10,000	56 % 150,000 to 200,000	
4 % 10,000 to 12,000	60 % 200,000 to 300,000	
5 % 12,000 to 14,000	63 % 300,000 to 500,000	
6 % 14,000 to 16,000	64 % 500,000 to 1,000,000	
7 % 16,000 to 18,000	65 % over 1,000,000	
. . . .		

Find the income tax of Mr. Royce, who has a net income of $ 9800 and is the father of three children.

Income	$ 9800	Surtax $ 5000 to $ 6000 = $ 10.00
Exemption	2600	Surtax $ 6000 to $ 7200 = 24.00
Taxable	$ 7200	Total surtax $ 34.00

4 % on $ 4000 = $ 160.00
8 % on $ 3200 = 256.00
Total surtax = 34.00
Total tax = $ 450.00

176. Inheritance Tax.—The federal government also taxes inheritances. The following gives a few of the rates: $50,000 exempt; $50,000 to $100,000, tax of 2 %; $100,000 to $200,000, tax of 4 %; $200,000 to $300,000, tax 6 %; gradually increasing scale up to $10,000,000, above which the tax is 25 %. Most states have an inheritance tax.

EXERCISES

1. The sales of a merchant (single) amounted to $16,500 for the year. He paid $11,400 for goods and $2700 for all other expenses. What was his net income? What income tax must he pay?

2. A farmer (married, having three children) sold live stock for $4300, hay for $1850, and grain for $350. He paid $685 for help, $349 taxes, $650 incidental expenses, and $315 interest on money borrowed. What was his net income? What was his income tax?

3. A consulting engineer (single) earned a yearly salary of $4300 and received $245 royalty on a patent. His travelling expenses in earning his salary were $385 and he had to pay his attorneys a fee of $125 for protecting his patents. What was his net income? income tax?

4. Show that the person with the larger net income can readily pay a higher rate of income tax than the person with a smaller net income.

5. What income tax will be paid on the following net incomes: $3400 (single)? $1900 (married, one child)? $1600 (single)? $3500 (married, three children)?

6. What will be the inheritance tax on the following amounts: $85,600? $35,000? $135,000?

7. Graph the surtax for incomes up to $18,000. What is the surtax on an income of $5550? $11,500? $7500?

177. Duties, or Customs.—A tax, called either **customs** or **duties,** is levied upon certain imports. The tax on an imported article is added to its price and is thus paid by the consumer. Customs are of two kinds: **specific** and **ad valorem.** Specific duties are certain amounts for each article or definite quantity of the same. Ad valorem duties are per cents of the cost. The list of duties levied upon various articles imported is called a **tariff.** This name is also given to the duties themselves.

178. Computing Duties.—In finding the duties on any import the valuation in foreign money must first be reduced to United States money, if the duty is ad valorem. If the duty is specific, the quantity, if expressed in some unit of measure other than the English unit, must be reduced to the latter. The units other than the English that arise are the metric, which are definitely related to the English units. Hence, such changes can easily be carried out. The ratios of money values vary from time to time according to the rate of exchange. The United States customs officials state from time to time the ratio of value of the dollar to the money units of the various other countries.

179. Tariff Illustrations.—The following are a few items taken from a recent United States tariff:

Sugar candy—below 15 ¢ per pound.............1 ¢ per pound.
Sugar candy—above 15 ¢ per pound.............25 % ad val.
Rice.......................................1 ¢ per pound.
Hay..$ 2 per ton.
Pineapples.................................$ 5 per 1000.
Cotton handkerchiefs.......................30 % ad val.
Woollen clothing...........................35 % ad val.
Gloves (leather)...........................$ 1 to $ 4.75 per doz.
Toys.......................................35 % ad val.
Wool.......................................8 % ad val.

•

EXERCISES

In the problems below use the tariff on the opposite page and wherever necessary use the rates of exchange found on page 110.

1. Find the total duty on 350 lb. sugar candy at 13 ¢ per pound and 250 lb. candy at 23 ¢ per pound.

2. What will be the duty on an importation of 3000 doz. Cuban pineapples? Hawaiian pineapples?

3. A manufacturer of woollen goods imported from England 60 yd. at 4 s. per yard, 120 yd. at 5½ s. per yard, and 80 yd. at 4½ s. per yard. Find the total cost of the importation and the total duty.

4. What will be the cost in United States money per yard of each grade of cloth mentioned in Ex. 3 after adding the duty?

5. Express by an equation the total cost of d dozen pairs of gloves bought at £ P per dozen with a duty of \$ U per dozen. What is the cost per dozen pairs? per pair?

6. Find the total cost of 150 doz. pairs of leather gloves costing £ 5 per dozen if the duty is \$ 3 per dozen pairs. What is the cost per dozen pairs? per pair?

7. What will be the duty on 7 metric tons of rice?

8. If the rice in Ex. 7 was bought at 205 yen per metric ton, what was its value in United States money?

9. After adding the duty, what would be the price per pound of the rice in Ex. 8?

10. Find the duty and total cost of 86,500 Kg. wool from Argentine bought at 2.13 pesos per kilogram.

11. What will be the duty on an importation of toys valued at 2450 francs? Give derivation of "ad valorem."

XI

TRANSPORTATION

180. Problems of Transportation.—Why is the cost of transportation an important item in our daily expenses? Why was this not so 50 yr. or 100 yr. ago?

A city having several railroads entering it has clay deposits of excellent quality just outside for making bricks. The oil used for baking the bricks must, however, be shipped in. Would it be more economical to ship the clay to a place where oil is found and there make the bricks? Why? A pottery was established in the same city. Both pottery clay and fuel had to be shipped in. The pottery failed. Why?

181. Railroads in the United States.—On July 4, 1828, the first rail of the Baltimore and Ohio road was laid. How long ago was that? Since then the increase in railroad mileage in the United States has been steady. This increase, together with our railroad mileage, that of Europe, and of the world for 1910, is found below:

1830.........	23 mi.	1880........	93,296 mi.
1840.........	2,818 "	1890........	163,587 "
1850.........	9,021 "	1900........	193,346 "
1860.........30,635	"	1910........	240,000 "
1870.........52,914	"	1918........	313,000 "

Europe ....202,000 mi.
United States . . . .240,000 "
World ....600,000 "

182. Railroad Investments.—Costs of railroad tracks and equipments vary so greatly that they can only be estimated roughly. Tracks and right of way cost from $ 30,000 to $ 220,000 per mile; engines from $ 20,000 to $ 40,000; passenger-cars about $ 12,000; sleeping-cars about $ 30,000; freight box-cars about $ 1500; open coal-cars about $ 1200.

EXERCISES

1. Make a graph showing the increase in railroad mileage in the United States. From the graph estimate the mileage in 1855; in 1905; in 1925. If the graph shows any special peculiarities, explain them as far as possible.

2. Estimate what per cent of the total railroad mileage of the world the United States had in 1910. Next compute the per cent and compare with your estimate.

3. How many cars were there in some freight and passenger trains that you have seen ? Using the above estimates, give a rough idea of the values of these trains.

4. Estimate the value of the track and right of way of the railroad from your city to some other at $ 50,000 per mile.

5. Let some member of the class look up and report on the building of the Baltimore and Ohio railroad. Solve some of the quantitative questions the report raises.

6. Consider the Union Pacific railroad similarly.

183. Passenger Fares.—Except where the cost of transportation is very high, as in the mountains, the passenger fare in the United States is 3 ¢ per mile. See railroad schedules for cost of a standard sleeping-car berth. Upper berths cost 80 % of lower berths. Berths in tourist sleepers cost 50 % of the corresponding berths in standard sleepers.
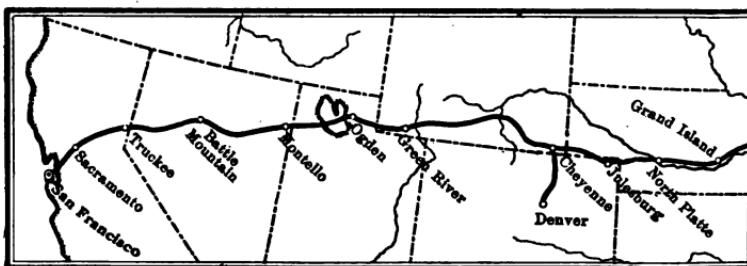
READ DOWN			SOUTHERN PACIFIC	‚READ UP	
10	20	MILES	DESTINATION	19	3
11 40	10 20	0	.....San Francisco..P.T.	8 50	1 30
3 30	1 40	89	.....Sacramento	5 20	8 05
10 40	7 43	208	.... Truckee	11 55	2 40
8 15	3 21	476	....Battle Mountain ...	3 30	4 27
2 40	8 30	662	.......Montello	10 05	10 15
7·00	11 55	782	.... Ogden ....P.T.	6 05	5 40
			UNION PACIFIC		
8 35	1 10	782	........Ogden....M.T.	6 50	6 15
3 30	7 15	959	.....Green River......	1 15	.11 40
2 15	5 20	1267	......Cheyenne.......	3 35	11 20
5 20	5 20	1267	......Cheyenne.......	1 15	10 55
8 55	8 55	1374	........Denver........	10 00	7 20
2 30	5 35	1267	......Cheyenne.......	3 20	9 45
6 27	.....	1414	......Julesburg......	10 18	3 24
8 50	11 10	1495	.....North Platte.M.T.	7 50	12 30
9 50	12 15	1495	.....North Platte..C.T.	8 50	1 20
1 35	3 50	1632	..... Grand Island	4 25	8 50
5 30	7 35	1786	........Omaha....C.T.	12 30	4 30

P.M., bold-faced.   A.M., light-faced.

EXERCISES

Use the rates in Art. 183 and the schedules on pages 142, 143 in solving the following exercises:

1. How can train No. 3 on the Union Pacific arrive at North Platte at 1:20 and leave at 12:30? Where else in these schedules is the same change found?

2. What is the fare from San Francisco to Pittsburgh, including $ 9.50 for tourist sleeper?

3. What is the fare from Omaha to Julesburg? from Youngstown to Baltimore?
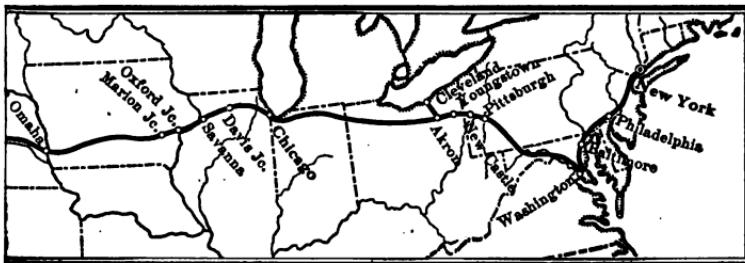
CHICAGO. MILWAUKEE AND ST. PAUL					
READ DOWN				READ UP	
20	26	MILES	DESTINATION	19	3
7 50	7 42	1786	........Omaha ....C.T.	12 16	3 25
3 10	3 12	2050	.......Marion Jc......	4 15	5 30
4 06	4 11	2086	......Oxford Jc.......	3 22	4 06
5 25	5 40	2140	.......Savanna........	2 00	2 10
7 00	7 05	2198	........Davis Jc......	12 42	12 23
9 15	9 00	2278	.......Chicago ...C.T.	10 45	9 50
BALTIMORE AND OHIO					
6	14	MILES	DESTINATION	55	5
5 45	9 30	0	...... Chicago ...C T	8 15	8 50
2 34	7 20	352	........Akron........	9 50	12 19
.....	8 50	352	........Akron........	9 20	.....
.....	9 45	389	.......Cleveland......	8 20	.....
2 40	7 32	352	...... Akron........	9 45	12 17
3 58	8 58	406	..... Youngstown	8 06	11 07
4 30	9 45	425	.....New Castle...C.T.	7 35	10 41
5 35	10 50	425	.....New Castle...E.T.	8 30	11 38
8 10	1 15	484	......Pittsburgh......	6 35	9 45
4 45	10 30	787	......Washington.....	9 10•	1 25
5 50	11 30	827	.......Baltimore......	8 00	12 25
8 19	3 50	923	..... Philadelphia.....	4 15	10 17
10 50	7 00	1013	...... New York ..E.T.	11 50	7 50

4. How long will it take to go from Sacramento to Ogden? Chicago to Washington? Philadelphia to Denver?

5. What train would you take and why in going from Ogden to Cheyenne? from North Platte to New York? from Denver to Omaha? from Akron to Baltimore?

6. By the use of railroad time schedules plan a trip from your city to some other, giving distance, time, and expense.

184. Express Rates.—Every express agent has a large book containing about three hundred schedules like those marked 44, 48, etc., below, which he uses in computing express charges. This book also tells him what schedule to use for each express office in the United States. It further classifies the various articles the company will carry.

Schedule Number	44		48		66		75		
	class		class		class		class		
lb.	1	2	1	2	1	2	1	2	lb.
1	30	30	31	31	32	32	32	32	1
3	35	35	36	36	40	40	41	41	3
5	41	41	42	42	47	47	49	49	5
8	49	41	50	42	58	49	63	54	8
10	54	41	56	43	66	49	71	54	10
15	68	52	71	54	86	65	93	70	15

The following are a few classifications:

Eggs—for hatching, 1; for table use, 2.

Fruit—open crates, 1; closed crates, 2.

Nuts—2.

Poultry—2.

Vegetables—open crates, 1; closed crates, 2.

Printed matter valued at less than $ 10—3d class at 8 ¢ per pound, with a minimum of 15 ¢.

All express valued below $ 50 is fully insured without charge. Above that amount the insurance is 10 ¢ per $ 100. Large sums of money and valuable papers are usually sent by express fully insured.

EXERCISES

1. Find the cost of sending 10 lb. nuts to a city in schedule 75.

2. Find the cost of sending 15 lb. strawberries in a closed crate to a city in schdeule 66. How much is saved by using a closed in place of an open crate ?

3. What will it cost to insure each of the papers sent by express having the following valuations: $ 500 ? $ 3250 ? $ 1475 ? $ 5650 ? $ 1780 ? $ 34 ? $ 45,500 ?

4. A business man had to take a long trip. At the end of his trip he would need some of his valuable documents. How could he get the documents for use without the risk of carrying them on the trip ?

185. Freight Rates.—Express is sent on passenger or special trains at 30 mi. to 50 mi. per hour while freight-trains go at about 12 mi. per hour. Express companies collect and deliver goods sent through them, while shippers and receivers of freight must bring it to and from the depot or warehouse, except in special cases where the cars are delivered upon a private switch at the warehouse of the receiver.

Freight agents have schedules quite like the express schedule mentioned on the last page. Look on the bill of lading found on page 146 for a list of the various freight classifications. Freight rates are quoted upon the hundred pounds in place of upon the pound as in express.

In cities a charge of from 40 ¢ to 50 ¢ per ton is made for taking cars to some particular track. A **demurrage charge** is also assessed against cars not unloaded promptly at the rate of $ 2 per day for the first four days and $ 5 per day thereafter. Why this difference in rates ?

EXERCISES

1. What will it cost to ship 5460 lb. household goods if the rate is 87 ¢ per hundred ?

2. Find the cost of shipping 960 lb. flour at 68 ¢ per hundred.

Uniform Bill of Lading—Standard form of Straight Bill of Lading approved by the Interstate Commerce Commission by order No. 787 of June 27, 1908, including provisions to conform with the requirements of the Commission Amendment to the Act to Regulate Commerce, effective June 2, 1918.

UNITED STATES RAILROAD ADMINISTRATION
Director General of Railroads

Atchison, Topeka and Santa Fe Railroad

STRAIGHT BILL OF LADING—ORIGINAL—NOT NEGOTIABLE.

Shipper's No. _____

Agent's No. _____

RECEIVED, subject to the classifications and tariffs in effect on the date of issue of this Original Bill of Lading

at _____ 19___

from _____

The Rate of Freight from _____

to _____ is in cents per 100 lbs.

If Towns 1st	If 1st Class	If 2d Class	If 3d Class	If 4th Class	If 5th Class	If A Class	If B Class	If C Class	If D Class	If E Class

Consigned to _____

Destination _____ State of _____ County of _____

Route _____ Car Initial _____ Car No. _____

NO. PACKAGES	DESCRIPTION OF ARTICLES AND SPECIAL MARKS	WEIGHT (Subject to Correction)	CLASS OR RATE	CHECK COLUMN

If charges are to be prepaid, write or stamp here, "To be Prepaid"

Received $_____ to apply in payment of the charges on the property described hereon.

Charges Advanced:
$_____

Shipper _____   Agent. _____

Per _____   Per _____

(THIS BILL OF LADING IS TO BE SIGNED BY THE SHIPPER AND AGENT OF THE CARRIER ISSUING SAME.)

186. Bill of Lading.—The freight agent in accepting freight to be sent over his road makes out a **bill of lading,** as shown above. It states the kind of freight, the rate, the total charges, the **consignor** (sender), the destination, and the **consignee** (to whom sent). Receipts for freight and for *payment* of charges are also given to the consignor. The

bill of lading is sent to the consignee, who presents it to the freight agent when claiming the goods.

In addition to the above-described bill of lading, also called **straight bill of lading,** there is the **order bill of lading.** This does not contain the name of the consignee, which is written in later by the consignor or by his agent. Suppose that Mr. Jones buys a car of coal for which he is to pay cash. The coal company sends an order bill of lading to a bank in Mr. Jones's city which turns the bill of lading over to him upon his payment for the coal.

EXERCISES

1. If you sent a box of nuts to one of your cousins in another city, what would you look for upon the bill of lading? What would you do with the bill of lading? How would your cousin obtain the nuts from the freight agent?

2. J. A. Howe shipped a car of 28,600 lb. of hay to W. W. Royde on an order bill of lading. The hay cost $ 12.50 per ton and the freight charges were 17 ¢ per 100 lb. What was the procedure for Mr. Royde to obtain the hay? How much did it cost him, including the freight?

3. Find the cost of sending a car of 645 bu. corn, 56 lb. to the bushel, at $14\frac{1}{2}$ ¢ per 100 lb.

4. Freight-cars are built to hold a certain capacity. How many bushels of oats, corn, or wheat can be shipped in a car with a capacity of 40,000 lb.? of 30,000 lb.? of 60,000 lb.?

5. Notice the capacity marked on some freight-cars. How many bushels would each hold of oats? of corn? of wheat?

6. A car of cattle weighing 34,600 lb. is shipped to market. What is the freight at 19 ¢ per hundred?

187. Inter-Road Charges.—The standard railroads in the United States have the same width of tracks. What advantage is there in this? If a railroad carries freight over its lines in a car belonging to another road, it pays this other road 25 ¢ for each day the car is used and $\frac{3}{4}$ ¢ to 1 ¢ for each mile it is hauled.

<center>EXERCISES</center>

1. A freight-car belonging to the New York Central Lines is used by the Burlington Road at $\frac{3}{4}$ ¢ per mile from Chicago to Denver—1034 miles—and return. The trains hauling the car average 12 miles per hour and the car remains in Denver 4 da. before being returned. How much is the Burlington Road indebted to the New York Central Lines?

2. Carloads of what commodities are sent out from your city? To what city will they likely be sent? How far is this? How long a time will it take? If the shipment is not made in a car belonging to the road hauling it, what would be the charge by the road owning the car?

188. Telephone and Telegraph Rates.—Local telephone rates vary greatly. Costs of long-distance calls depend upon distance and length of time of talking.

The cost of a telegram depends upon its length, distance sent, and time of day it is sent. **Regular messages** are quoted for 10 words and each additional word. A 50-word **day letter** costs 1.5 times the regular message, but is delivered any time during the day. **Night messages** usually cost less than the regular message and are delivered the morning after they are sent. **Night letters** of 50 words cost the same as the regular message and are delivered the following morning. Why this variation?

EXERCISES

1. Write a 10-word telegram that you will arrive in Denver over the Burlington R. R. Wednesday at 10:40 a.m.

2. Condense Art. 188 into a 50-word letter. If a 10-word message cost 50 ¢, what will be the cost of a day letter? of a night letter?

3. Condense Ex. 1, page 148, into the briefest possible telegram. Find the cost at 48 ¢ for first 10 words and 3.5 ¢ for each additional word.

4. What are the telephone rates in your city? Can you find the rates in some other city?

189. United States Mail.—The rates on United States mail are as follows:

First Class.—Post-cards: 1 ¢ in United States and possessions; Mexico; Canada; and Shanghai, China. Sealed letters, 2 ¢ per ounce, same as above; 5 ¢ for first ounce and 3 ¢ each additional ounce to foreign countries. Post-cards 2 ¢ to foreign countries.

Second Class.—Newspapers, 1 ¢ for each 4 oz.

Third Class.—Miscellaneous printed matter, 1 ¢ for each 2 oz.

Fourth Class.—*a.* Parcels weighing 4 oz. or less, except books, plants, and so on, 1 ¢ for each ounce, any distance. *b.* Parcels weighing 8 oz. or less containing books, seeds, cuttings, bulbs, roots, scions, and plants, 1 ¢ for each 2 oz. or fraction thereof, any distance. *c.* Parcels weighing more than 8 oz., like the above; printed matter weighing over 4 lb.; and all other fourth-class matter by rates shown on the following page.

EXERCISES

1. What will be the cost of sending a letter weighing $3\frac{1}{4}$ oz. to a place in the United States?

2. What will be the cost of sending the above letter to France? to Mexico? to China?

3. What will be the cost of sending to places in the United States: 3 letters, each weighing $\frac{3}{4}$ oz.; 2 papers, 5 oz. each?

PARTIAL TABLE PARCEL-POST RATES

Weight in Pounds	Local	ZONES							
		1st Up to to 50 miles	2d 50 to 150 miles	3d 150 to 300 miles	4th 300 to 600 miles	5th 600 to 1000 miles	6th 1000 to 1400 miles	7th 1400 to 1800 miles	8th Over 1800 miles
1	$0.05	$0.05	$0.05	$0.06	$0.07	$0.08	$0.09	$0.11	$0.12
2	.06	.06	.06	.08	.11	.14	.17	.21	.24
3	.06	.07	.07	.10	.15	.20	.25	.31	.36
4	.07	.08	.08	.12	.19	.26	.33	.41	.48
5	.07	.09	.10	.14	.23	.32	.41	.51	.60
10	.10	.14	.14	.24	.43	.62	.81	1.01	1.20
15	.12	.19	.19	.34	.63	.92	1.21	1.51	1.80
20	.15	.24	.24	.44	.83	1.22	1.61	2.01	2.40
25	.17	.29	.29	.54	1.03	1.52	2.01	2.51	3.00
30	.20	.34	.34	.64	1.23	1.82	2.41	3.01	3.60
35	.22	.39	.39	.74	1.43	2.12	2.81	3.51	4.20
40	.25	.44	.44	.84	1.63	2.42	3.21	4.01	4.80
45	.27	.49	.49	.94	1.83	2.72	3.61	4.51	5.40
50	.30	.54	.54	1.04	2.03	3.02	4.01	5.01	6.00
60	.35	.64	.64	1.24					
70	.40	.74	.74	1.44					

Parcels may be insured upon payment of a fee of 3 cents for value not exceeding $ 5, 5 cents for value not exceeding $ 25, 10 cents for value not exceeding $ 50, or 25 cents for value not exceeding $ 100.   It may not be registered.

Limit of Weight.—70 lb. in first three zones, and 50 lb. in other zones.
Limit of Size.—Length and girth combined may not exceed 85 in.
War Tax.—1 cent for each 25 cents or fraction thereof.
For more complete statement, apply to your postmaster.

EXERCISES

What will be the cost of sending:

1. 25 lb. local ?

2. 34 lb. 7 oz. for 160 mi. ?

3. *45 lb. for 960* mi. ?

4. 20 lb. for 325 mi. ?

5. 14 lb. 5 oz. for 580 mi. ?

6. 60 lb. local ?

7. On an outline map, or some other map, of the United States with your city as centre, draw circles having radii that correspond to distances for the various zones.

8. Select six cities in the United States and note in what zone they are located. What will it cost to send 14 lb. 9 oz. of merchandise to each of these cities ?

9. Select a city in each of the first, fourth, and seventh zones. Find the cost of sending to each city by parcel post merchandise weighing 1 lb.; 5 lb.; 40 lb.; 50 lb.

10. Select two or three members of your class to find the cost of sending these articles by express.

11. Select two or three members of your class to find the cost of sending these articles by freight.

12. Compare the costs as found for Exs. 9, 10, and 11. Which would you likely use ? Why ?

13. Note from the table on the opposite page that the rate in cents for the 3d zone is 2 times the weight in pounds plus 4. State the total cost, C, of sending a package within the 3d zone in terms of its weight in pounds, P.

14. Find the cost of sending to a city in the 3d zone a parcel weighing 13 lb.; 32 lb. 5 oz.; 28 lb. 7 oz.

15. Try to discover the law of rates in some other zone. When you are certain that you have found it, express it by an equation, as in Ex. 13.

16. Henry found that eggs averaged 2 oz. in weight which he sent to his uncle living in the 3d zone. What will it cost to send 5 doz. in a crate that weighs 2 lb. 3 oz. ?

17. Express the limit in size of cylindrical packages in accordance with that on page 150. Can a cylindrical package 4 ft. long with a radius of 5 in. be sent parcel post ?

190. Good Roads.—It is estimated on good authority that 5,000,000,000 tons are hauled yearly over the roads of the United States. The average distance each ton is hauled is 9.4 mi. The amount of hauling needed, say, to fill a bin with coal depends upon the number of tons hauled and upon the distance they were hauled. It will cost the same to haul 20 T. for 1 mi. as 1 T. for 20 mi., or 5 T. for 4 mi. To find the amount of hauling multiply the number of tons by the distance in miles. The result is called **ton-miles.** Explain the meaning of this.

In France the average cost of hauling upon its good roads is 10 ¢ per ton-mile. The average cost in the United States is 23 ¢ per ton-mile. Why this great difference? It is estimated that with good roads here the cost can be reduced to the same as that in France. Why are the roads in the United States not as good as in France?

EXERCISES

1. By the use of 10 and the proper exponent express simply the number of tons hauled over our roads yearly.

2. From Art. 190 find the number of ton-miles goods are hauled yearly in the United States.

3. What is the yearly cost of hauling the goods, as given in Ex. 2, at the average cost of 23 ¢ per ton-mile?

4. What would be the price of hauling the goods, as given in Ex. 2, at 10 ¢ per ton-mile?

5. How much would be saved yearly by having better roads which would decrease the cost per ton-mile to 10 ¢?

6. The Panama Canal cost $ 375,000,000. How many times this value is lost yearly on poor roads?

7. What will it cost a farmer living 6 mi. from market to deliver 1500 bu. wheat, 2000 bu. oats, and 17 T. hay at the rate of 23 ¢ per ton-mile? 10 ¢ per ton-mile?

8. In 1913 it cost the farmers 250 million dollars to haul to market their crops valued at 10 billion dollars. Express each number in numerals. What per cent of the value of the crop was used in hauling it to market?

9. If the population in the United States was 100,000,000 in 1913, what was the cost per capita in marketing the crops for that year?

10. If the cost of marketing by the farmers had been reduced 55 % by good roads, what would have been saved that year?

11. The United States has about 2,000,000 mi. of roads. It is estimated that 80 % of the hauling is done over 25 % of the roads. What would this part of the cost of the hauling be for 1913 and over how many miles?

12. If this 25 % of the total road mileage of the United States were improved so as to save 55 % of the cost of hauling on them, what would be the saving upon these roads? What would be the saving per mile?

13. A certain county in one of the central states has 270 mi. of roads. One year this county paid $ 2220.24 upon its roads. How much is this per mile?

14. If possible, find the data for your county or for some county in your state to answer the question of Ex. 13.

15. The value of the farms in the United States is estimated at $ 40,000,000,000. Read this number and express it in the simplest exponential form.

16. Why is improvement of roads equivalent to bringing the farms nearer to market? How does the value of a farm near to market compare with one farther from market? It has been estimated that good roads would increase farm values 25 %. Express this in dollars by use of Ex. 15.

191. Water Transportation.—Freight can be sent by water much more cheaply than by rail. It has two drawbacks: waterways are not always available and the time of transit is always longer. Explain these effects upon traffic.

192. Waterways of the United States.—The United States possesses the following waterways:

> 25,000 mi. non-navigable rivers.
> 25,000 mi. navigable rivers.
> 1,410 mi. lakes.
> 2,120 mi. canals.
> 2,500 mi. Atlantic and gulf coast.
> 3,500 mi. possible canals to connect certain of the above
> —future construction.

193. Tonnage of Waterways.—In 1906 the following approximate tonnage was carried on the U. S. waterways:

> 75,610,000 T. Great Lakes and St. Lawrence.
> 19,530,000 T. Mississippi River.
> 6,527,000 T. Hudson River.
> 3,717,000 T. all others.

194. Tonnage.—The ton, in shipments within the United States, is 2000 lb., but to and from foreign ports it is the long ton of 2240 lb. or the metric ton. **Tonnage** is given in the capacity of 40 cu. ft. for each long ton, based upon the space occupied by a long ton of wheat. The measurements and registering of the tonnage of boats and ships are quite technical and will not be given here.

The value of our imports and exports in 1908 was $3(10)^9$. The tonnage for the same year was 19,000,000 T. imports and 51,688,148 T. exports.

<div align="center">EXERCISES</div>

1. Find the average value per ton of imports and exports for the year 1908.

2. The imports are what per cent of the exports?

3. The exports are what per cent of the imports?

The first of the numbers below each city gives the distance in miles from New York by the old water-route. Below this is the distance by way of Panama Canal.

San Francisco	Yokohama	Shanghai	Sydney
13,714	13,564	12,514	13,658
5,299	9,835	10,885	9,814

4. Find for each city above the per cent of the old distance saved by going through the Panama Canal.

5. A lake transport can make 9 return trips yearly from Chicago to New York. If it can carry 2200 T. each trip, how many tons can it carry in all?

6. The total yearly cost during one year for such a transport as that in Ex. 5 was $65,540. What would that make the cost of freight between Chicago and New York per ton? per C?

7. It cost a transport $63,990 during one year in running from Chicago to Buffalo. It carried that year 70,400 T. What was the cost of transportation per ton? per C?

8. If possible, ascertain the railroad freight rates corresponding to the above and compare.

9. Use literal numbers to answer questions in Exs. 1, 2, and 3 for the general case by means of an equation.

10. If possible, ascertain recent data to substitute in equations formed for Ex. 9.

11. An English ship charged $2\frac{1}{2}$ d. per bushel for carrying wheat from New York to Glasgow. What would be the freight in United States money on 2400 tons of wheat?

12. Name two important foreign waterways.

XII

WAGES

195. Forms of Wages.—Compensation for service in the form of salary or wages is agreed upon in many various ways. Managers and their direct assistants are usually paid by the year or by the month. These men give all of their time to the work, not thinking of the number of hours per day devoted to it. Their pride and satisfaction in making the enterprise succeed is a large part of their compensation. The pay in some occupations is by the month of 26 da.; in others it is by the day of from 7 hr. to 10 hr., with overtime and holidays paid for at the rate of 1½ times the actual time worked. Factories, stores, and other establishments use time-clocks for recording upon individual time-cards when the employee enters and leaves. See opposite page. Usually any one late ¼ hr. or less loses ¼ hr.; one late between ¼ and ½ hr. loses ½ hr.; and so on. Any one checking out early loses time in the same manner.

No. 115.
Name Henry Q. Mace.

	Morning		Afternoon		Hr.
	in	out	in	out	
Mon.	7:57	12:04	12:56	5:07	8¼
Tue.	8:04	12:03	12:58	5:13	8¼
Wed.	7:54	12:07	12:57	5:05	8¼
Thur.	8:13	12:04	12:58	5:05	7¾
Fri.	7:56	12:13	1:06	5:08	7¼
Sat.	7:55	12:03	12:58	5:12	8
					47¾

	Hours	Rate	Amount	
			$	Cts.
Straight Time	47¾	40	19	00
Over Time	0			0
Total			19	00

156

Professional men make charges depending upon the nature of services rendered and the time required. Factories and other establishments also often pay for "piece work." The compensation paid for services rendered by agents will be discussed in the next chapter, "Commission and Brokerage."

EXERCISES

1. Will John make more money by working at the rate of $ 35 per month or $ 1.50 per day ?

2. If John, working at the above rate per day, misses 4 da. one month, how much should he be paid ?

3. In a certain city carpenters are paid 80 ¢ per hour. They work 8 hr. per day except Saturdays, when they work only 4 hr. What is the pay per week ?

4. How much time is lost by the owner of the time-card, page 156 ? If the pay is $ 18 per week for a 9-hr. day, what is the loss in money ?

5. A manager of a small manufacturing plant receives a yearly salary of $ 1800. He estimates that he works 12 hr. per day except Sundays. What is he paid per hour ?

196.   Time-Sheets.—Below is found a portion of a time-sheet for an eight-hour day, with $1\frac{1}{2}$ time for overtime. These are made weekly from the time-cards of each employee or the timekeeper's record, or it may be the very record sheet used by the timekeeper.

No.	Name	Hours per day						Time			Overtime			Total
		M	T	W	T	F	S	Hrs.	Rate	Amt.	Hrs.	Rate	Amt.	
1	H. Jones	9½	8½	8		8½	9	4	44	42	18.48	3½	63	2.20	20.68
2	F. Mayle	9½	8½	8		8½	8	4	44	45	19.80	2½	67	1.67	21.47
3	Y. Cone	9	7¾	7¾	8		8	4	43½	40	17.40	1	60	.60	18.00
4	V. Lore	9½	9	8	9	8	4								
5	Q. Love	9½	9	8½	8½	7½	4								

Make a graph showing wages at 40 ¢ per hour.   Make the graph so as to include as many hours as possible and still make it accurate.   Can you place graphs of other rates to the same axes?   Note how the latter shows the variation in incomes.

For piece-work pay-rolls are made out, based upon amount of work accomplished.

197.   Pay-Sheets.—Wherever payment is by **"envelope,"** that is by cash, **pay-sheets** like the following are made out. These tell not only the change to go into each pay envelope but also how much money of each denomination should be obtained from the bank.   How?

No.	Name	$10	$5	$2	$1	50c.	25c.	10c.	5c.	1c.	Total
1	H. Jones	2				1		1	1	3	$20.68
2	F. Mayle	2			1		1	2		2	21.47
3	Y. Cone	1	1	1	1						18.00
4	V. Lore										
5	Q. Love										

198. Overhead Expenses.—Factories, large stores, and other large mercantile establishments are divided into departments. A strict account is kept of the amount paid out for labor in each department. In addition to those giving all of their time to any one department there are usually several, as superintendents, who divide their work among departments. Salaries paid to such men are called **"overhead expense."** These are either charged to the concern as a whole or divided pro rata among the departments.

EXERCISES

1. Complete the computations for the time-sheet on the opposite page.

2. Complete pay-sheet at the bottom of the opposite page.

3. Make out the time-sheet and the corresponding pay-sheet for the following, the working-day being 8 hr.:

A. K. Jacks: M 7; Tu 8; W $8\frac{1}{2}$; Th $7\frac{1}{2}$; F $7\frac{1}{4}$; S 4.   65 ¢.

W. Q. Force: M 8; Tu 8; W 8; Th $8\frac{1}{2}$; F 8; S 4.   55 ¢.

G. Y. Mace: M $7\frac{3}{4}$; Tu 8; W 9; Th $7\frac{1}{2}$; F 8; S 4.   60 ¢.

R. V. Soles: M $6\frac{1}{2}$; Tu 8; W $9\frac{1}{2}$; Th $7\frac{3}{4}$; F 8; S $3\frac{1}{2}$.   65 ¢.

A. Z. Jose: M $7\frac{1}{4}$; Tu $7\frac{3}{4}$; W $8\frac{1}{2}$; Th $7\frac{3}{4}$; F $6\frac{1}{2}$.   65 ¢.

E. U. Zose: M 8; Tu $8\frac{1}{2}$; W 9; Th $8\frac{1}{2}$; F 8; S 4.   55 ¢.

Y. T. Stone: M $7\frac{1}{4}$; Tu 8; W 9; Th $7\frac{1}{4}$; F 8; S $3\frac{1}{2}$.   60 ¢.

M. K. Loat: M $7\frac{3}{4}$; Tu 8; W 8; Th $7\frac{3}{4}$; F $7\frac{1}{2}$; S $3\frac{1}{4}$.   65 ¢.

D. K. Royce: M $7\frac{1}{2}$; Tu 9; W $7\frac{3}{4}$; Th $7\frac{3}{4}$; F $8\frac{1}{2}$; S 4.   55 ¢.

4. A bricklayer in a certain city is paid 60 ¢ per hour, and a carpenter 50 ¢. If the bricklayer loses 25 % of the working time because of weather conditions, while the carpenter is idle only 10 % of the working time, which will receive the higher yearly wage ? Solve as simply as possible.

199. Value of Education.—The value of an education is not doubted to-day, but few persons take the trouble to actually compute its value in dollars and cents. The following tabulation was prepared by a committee from the Brooklyn (N. Y.) Teachers' Association. It shows only a part of the money value of an education, as wages after 25 yr. of age are not recorded.

Weekly income when	Left school at 14 yr.	Left school at 18 yr.
14 yr. of age	$ 4.00	
15 yr. of age	4.50	
16 yr. of age	5.00	
17 yr. of age	6.00	
18 yr. of age	7.00	$ 10.00
19 yr. of age	8.50	10.75
20 yr. of age	9.50	15.00
21 yr. of age	9.50	16.00
22 yr. of age	11.75	20.00
23 yr. of age	11.75	21.00
24 yr. of age	12.00	23.00
25 yr. of age	12.75	31.00

The same committee also found that of 12,000 who had only a very elementary education the yearly wage was $ 657. In another department requiring a high school education, the average yearly wage was $ 1597; while in one requiring a college education, the average wage was $ 2400.

The above is based upon data secured during normal times and will prevail during a long period of time. During abnormal times the above conclusions may not hold, but this is for only very short periods of time.

EXERCISES

Assume that the average man works 50 weeks per year. Find the following from the above table:

1. The difference in yearly wages at 18 yr.; at 20 yr.; at 25 yr.

2. How much has each·man earned at the age of 26 yr. ?
How much more has one earned than the other ?

3. What has been the yearly advance in income of each
for the five years from 20 yr. to 25 yr. ?

4. What was the per cent of rise in income of each for
these 5 yr. ?

5. Suppose that the same rate of advance in incomes
holds for the next five periods of 5 yr., that is, up to 50 yr.
of age, what will be the income of each ? How much will
one receive more than the other ?

6. How much money placed at 5 % yearly interest
would earn the difference in incomes at 25 yr. ? The mean-
ing is that the amount you have just found is the extra
capital obtained by the boy who remained in school from
14 yr. to 18 yr.

7. The boy who remained in school the four years from
14 yr. to 18 yr. earned this capital during these four years.
How much was this per year ? per week ? per day ? Does
it pay in dollars and cents to remain in school ?

8. Find the difference in incomes of the first two classes
mentioned in the paragraph below the table on page 160.
This is equivalent to the income from how large a sum in-
vested at 5 % interest ?

9. Answer the questions of Ex. 8 for the second and
third classes of men mentioned in the same paragraph.

10. Answer the questions in Ex. 8 for the first and third
classes of men mentioned in the same paragraph.

11. Graph the two changes in incomes found on page 160.
What significant facts can you deduce from the graph ?

12. Find the average yearly wages which for three years
are $ *a*, $ *b*, and $ *c* ? Find the average yearly wages for the
two cases mentioned on page 160 from 18 yr. to 25 yr.

XIII

COMMISSION AND BROKERAGE

200. . **Commissions.**—Many business transactions are carried out by agents. The compensations and the arrangements under which the agents work vary, not only with different commodities sold, but also in different localities. As a rule the compensation is a per cent of the sum involved in the transaction and is called **commission.** The names applied to the various classes of agents, together with the different commissions, are in general those given below.

1. **Real estate agents** often sell land at a stated amount per acre or any real estate for a certain per cent. Most states have a law fixing the rate of commission in case of a dispute. Some agents charge a higher rate for selling a cheap than a high-priced property. Why? The owner of a property may list it with the agent for a certain sum net; the agent then gets as his commission whatever he can sell the property for above this amount. Except in special cases all the fee is paid by the one selling the property. These agents also take care of property, keeping it leased, having taxes paid, repairs kept up, and so on for either a definite yearly sum or a per cent of the rent.

$$C = S \times R \,\%. \qquad (1) \quad \text{Explain.}$$

Also net returns to the owner are:

$$N = S - C = S - S \times R \,\% = S(1 - R \,\%). \quad \text{Explain.}$$

2. One selling articles as books, aluminum ware, and so on from house to house is generally called a **canvassing agent,** or merely a **canvasser.** The rates of commissions are high, sometimes being as much as 50 %.

3. The producers of certain articles, as some breakfast-foods, set the retail price and supply the dealer at a lower price. The difference in the two prices is really a form of commission. What other articles are sold in this manner?

4. **Travelling salesmen** for wholesale houses often receive a commission in addition to a definite salary.

5. **Retail clerks** in rare instances receive a commission in addition to the regular salary. The rate of commission is low: 1 % to 3 %.

6. **Employment agencies** help persons to secure positions. The charge is generally a definite amount varying with the position secured. Sometimes it is a per cent, as in the case of teachers' agencies, which charge 5 % of the first year's salary. For additional commissions see Arts. 201 and 202.

EXERCISES

1. How would you go about finding a buyer for a house, a lot, or land that you desire to sell?

2. What will be a real-estate agent's commission at 3 % for selling a house and lot at $ 5950 ? What will the owner of the house receive?

3. What is the legal rate of real-estate commission in your state? What would be an agent's commission at this rate for selling a house for you at $ 4650 ? What would you pay an agent if you bought a house that he had listed for $ 3500 ?

4. What is the commission on selling a house for $ 3500 at $2\frac{1}{2}$ % ?

5. What is the legal real-estate commission in your state on a house and lot sold for $ 3850 ? sold for $ 4700 ? sold for $ 4500 ?

6. What is the commission at 5 % on selling a lot for $ 600 ?

7. If the per cent of commission is known, how can the per cent of the sales price received by the owner be found? Show that this is $1 - r \%$.

8. Show that the net price to the owner is

$$N = (1 - r \%)S.$$

9. Solve the above equation for S.

10. In a certain city the real-estate agents charge a commission of $2\frac{1}{2}$ %. For how much must a house be listed in order to net $ 4500 after the commission is paid ?

11. What will be the commission for selling a half-section of land at $ 65 per acre if the commission is $2\frac{1}{2}$ % ? What will the owner of the land receive ?

12. When land is selling at $ 45 per acre which will net the best commission, $ 1.50 per acre or 3 % ?

13. For what must land be sold so that a 5 % commission and $ 2 per acre will be equal ?

14. What commission on a 9-mo. salary will a teacher pay who accepts through a teachers' agency a position that pays $ 150 per month ? $ 125 per month ? $ 185 per month ?

15. State by an equation the monthly wages received by a salesman, in terms of his fixed salary and of a per cent commission on all sales.

16. A travelling salesman receives a salary of $ 125 per month and a commission of $1\frac{1}{2}$ % on all sales. Find his income for a month in which his sales were $ 3570.

17. A travelling salesman is offered a salary of $ 225 per month or a commission of $2\frac{1}{2}$ % on all sales. Last year his sales averaged $ 10,500 per month. Which arrangement ought he to accept ?

201. Additional Forms of Commission.—

7. Men loaning money charge a yearly per cent of the money loaned. In one locality this commission is $\frac{1}{2}$ % per year. Hence, for 5 yr. this would be $2\frac{1}{2}$ % of the money loaned. How? Various arrangements are made as to the payment of this commission, but it is generally paid in advance when the loan is made.

8. **Law firms** and **collection agencies** collect debts for 10 % and up, depending upon the difficulty in making the collection.

9. Agents dealing in stocks and bonds (see Chapter XX) are called **brokers**. Their commission, called **brokerage,** is $\frac{1}{8}$ % of par value for selling and for buying shares of stocks and bonds. These transactions are carried out on the stock exchange of which the broker must be a member.

EXERCISES

1. Mr. Wiles sent three notes for $ 135.40, $85.60, and $ 245 respectively to his collecting agent. For collection of the first two he paid 10 % and for collection of the third 25 %. How much did the collecting agent remit to Mr. Wiles ?

2. Mr. Wilsmore loaned $ 500 to a reliable merchant at 6 % for one year and to another merchant he loaned $ 400 at 10 % also for one year. Find the amount of each note at the end of the year. Suppose that he had to pay $ 5 and 10 % commission for collecting the second note. What rate of interest did this pay him on his whole investment ? Which was the better investment ?

3. What will a broker charge for selling 6 Pike Township bonds with a par value of $ 500 each ?

4. What will be received for 7 city bonds, par value $ 100 each, if sold at $ 85$\frac{1}{8}$?

5. What will be paid for 6 C. and A. bonds sold at 96⅞ % of the par value, if par value of each bond is $ 1000?

6. What will the broker receive who bought and sold the bonds of Exs. 4 and 5 ?

202. Additional Forms of Commission.—

10. The produce **commission merchants** of the larger cities have fruit, vegetables, eggs, poultry, and so on sent to them to sell. Their commissions vary greatly. Why? The agent is sometimes called the **consignee** and the goods received a **consignment.** The person sending the goods makes a **shipment,** and he is called the **consignor.**

11. The various grains are bought and sold by the **grain commission merchants** on the **boards of trade** in the principal cities. They charge uniformly ⅛ ¢ per bushel for buying and the same for selling.

The **board of trade** is a company of men organized to carry on the grain trade in a city. They meet at their designated trading-room during stated hours daily, except holidays and Sundays. Each broker calls out what grain, how much, and at what price he has it for sale, or what grain, how much, and at what price he has orders to buy. They then carry out the various orders from their customers and afterward send the latter statements of the transactions. The prices of the various grains throughout any section of the United States become the prices attained on the board of trade in a near-by city, plus or minus the freight charges to that city, except as the prices may be regulated by the government. When would you add and when subtract the freight charges ?

12. Broom-corn is baled and sold by the ton on the *broom-corn* **exchange.**

13. Cotton is sold in bales of 500 lb. on the **cotton exchange.**

14. Live stock is sold on the **live-stock exchange.** At one such exchange the commissions on cattle are 60 ¢ per head up to a maximum of $ 15 per car-load. Cattle for feeding are bought for feeders at $ 12 per car-load. Hogs and sheep are sold at $ 10 per car-load. Show that these $ 10 and $ 12 are commissions.

EXERCISES

1. Mr. Jacks sent 14 cases of strawberries to his commission merchant who sold them for $ 1.85 a case. He charged 15 % commission for selling the berries, together with freight and drayage charges, $1.35. How much did the commission merchant remit to Mr. Jacks ?

2. Harry sent 3 crates of chickens that he raised during the summer to a city commission merchant to sell for him. The chickens weighed 168 lb. If they were sold at $19\frac{1}{2}$ ¢ per pound and the commission merchant charged 15 % for selling the chickens, how much did he remit to Harry ?

3. A car of wheat weighing 46,500 lb. is sold at $ 2.09 per bushel. What will be the commission for selling and what will be the net returns to the owner ?

4. A car of corn weighing 48,600 lb. is sold in Chicago at $58\frac{1}{4}$ ¢ per bushel. What will be the commission for selling and what will be the net returns to the owner ?

5. The expenses of a live-stock commission merchant are $ 345 per month. How many cars of cattle or cars of hogs must he sell per month in order to net him $ 200, which he could earn working for a firm in the same business ?

6. A canvasser receives $33\frac{1}{3}$ % commission. What will he receive for making each of the following sales: $ 12 ? $ 6.30 ? $ 14.45 ? $ 6.90 ? $ 8.40 ? $ 7.50 ? $ 9.20 ?

203. Sales Account.—After completing the sales of goods the agent sends to the owner of the goods, called **principal,** an itemized statement of the transactions, called a **sales account.** This should contain the sales, expenses, and net returns. The following is an illustration:

Chicago, Ill., June 10, 1918.

X. W. SAVER & SONS
COMMISSION MERCHANTS

SOLD FOR THE ACCOUNT OF

Henry Q. Restful,

Rayville, Ind.

```
June 3--30 cases strawberries--$ 2.75
  "  5--34    "        "        -- 2.40
  "  7--65    "        "        -- 1.85  _____
                   Charges.
June 4 Freight and cartage-----$ 7.40
  "  6    "       "        "   -----  6.30
  "  8    "       "        "   -----  8.65
       Commission 5%-------------      _____
                   Net Proceeds.
```

EXERCISES

1. Complete the above bill and find the net proceeds.

2. Make a sales account for the following: July 6, 15 T. hay at $ 12 per ton; July 8, 23 T. at $ 14.50; July 10, 19 T. at $ 17. Charges: Freight and drayage, $ 165.50; commission, $ 1.50 per ton. Sold by Jay C. Sase Co., Kansas City, Mo., for Jerry Civil.

3. Make a sales account for the following: Nov. 15, 112 lb. dressed poultry at 25 ¢ per pound; Nov. 22, 137 lb. dressed poultry at 26 ¢ per pound; Nov. 25, 215 lb. dressed poultry at 28 ¢ per pound. Charges: freight, $ 9.30; drayage, $ 2.50; *commission,* 5 % of sales.

204. Purchase Accounts.—These accounts are similar to the sales accounts except that they give the initial price, the expense, and the total or gross price.

EXERCISES

Write the correct sales or purchase accounts for the following, using your own and some other city, and make any necessary computations:

1. Henry Cracken sold for W. K. Lokes: May 8, 640 doz. eggs at 24 ¢ per dozen; May 15, 115 lb. poultry at 22 ¢ per pound; May 21, 89 lb. poultry at 21 ¢ per pound; May 29, 315 doz. eggs at 20 ¢ per dozen. The charges were: May 7, freight and drayage, $ 1.75; May 14, same, $ 2.25; May 20, same, $ 1.95; May 28, same, $ 1.35. Commission was at the rate of 5 %.

2. Bought and shipped to Wm. Q. Stout by the Mich. Commercial Buyers: Sept. 3, 250 boxes peaches at $ 1.85 per box; Sept. 11, 325 baskets grapes at 23 ¢ per basket; Sept. 17, 150 crates melons at 85 ¢ per crate; Sept. 28, 450 baskets grapes at 19 ¢ per basket. Commission, 8 %.

3. J. W. Colfer sold for the Western Milling Co. the following shipments of flour: Nov. 5, 140 bbl. at $ 16.50 per barrel; Nov. 11, 235 bbl. at $ 14.25 per barrel; Nov. 18, 85 bbl. at $ 15.40 per barrel. The charges were, freight and drayage to a total of $ 564.34. The commission charged was 4 %.

4. Make up items for a sales account of commodities shipped from your community and write the sales account.

5. From some dealer or from the library procure prices · of a certain commodity, as wheat or flour, at 3 mo. intervals for at least 5 yr. Graph your data. When and what was the highest price ? Account for variations shown in the graph.

XIV

EXPENSES

205. Need of Controlling Expenses.—It is not the man who draws the highest wages but the man who keeps expenses below his wages who accumulates means for future use. Similarly, it is not the business man who charges the highest prices nor who has the most business that makes the greatest profits. It is the one who keeps his total outlay·well below the income. This is also the business man who keeps his expenses down. In any enterprise the matter of expenses must be looked into very closely all the time.

Sometimes cutting down one expense increases another even more, or decreases efficiency too much. George saved $ 1 in material and 3 hr. of his time by not repairing the fence of the pasture where he kept his cow. The cow broke through and destroyed a neighbor's garden. George had to pay $ 5 for the damage to the garden. What examples can you give of a mistaken idea of saving?

206. Expenses Already Considered.—We have already considered several expenses connected with various enterprises. What are these?

207. Additional Expenses.—Many other expenses arise in every industrial and mercantile establishment. What additional expenses can you give connected with running a store? a hotel? a meat market? a bank? a farm? a fruit farm?

208. Buying Expenses and Producing Expenses.—Buying expense is the name given by large stores to the total cost of placing in their store or warehouse the goods they offer for sale. This would include salary to their buyers, buyers' travelling expenses, freight on goods, drayage, cost of sorting and of placing goods on shelves or in warehouses, and, in fact, every expense connected with getting the goods ready to sell. A manufacturing concern or a farmer would list under **expense of production** whatever it costs him to get his products ready for the market. For a factory it would be cost of raw materials, cost of all labor, overhead expense connected with production, pay to other factories for parts bought from them, transportation to the factory, rent, light, fuel, water, ice, interest on money invested, and all waste.

What are the principal buying expenses of the grocer? coal-dealer? milk-dealer? moving-picture theatre?

What are the principal expenses of production of a farmer? a street-car system? a school-teacher? a lawyer? a truck-driver?

209. Selling Expenses.—After the goods are in the store or warehouse the merchant calls all expenses connected with them **selling expenses.** Show that this would include rent, light, fuel, water, ice, interest on money invested, and waste. Expenses listed as selling expenses by all are wages of salesmen, losses on goods returned, losses by selling goods out of season, losses in depreciation, advertising, storage of goods ready to sell, cost of delivery of goods sold, and bad debts. Can you name any others?

What are the principal selling expenses of the farmer? grocer? coal-dealer? school-teacher? any manufacturing or business concern?

210. Rent.—A manufacturing plant that owns its buildings and grounds still has the expense of rent. If the buildings were not used, they could be rented or sold and the money from them invested at interest. Storage is another form of rent.

211. Other Running Expenses.—Light, fuel, phone, water, and ice are expenses common to all business and manufacturing concerns.

212. Delivery of Goods.—The farmer's heaviest selling expense is hauling his produce to the market. Deliveries are a great item of expense to the city merchant. In some cities the merchants have united and make their deliveries through a common delivery. Each merchant pays his part of the expense according to the number of deliveries made for him.

EXERCISES

1. A business firm uses its own building, which could be sold for $ 6500. If money can be loaned at 6 % and the insurance is 90 ¢ per $ 100, to what monthly rent would this amount ?

2. The reading of the Emporium Store's water meter was 230,100 gal. Sept. 1 and 245,800 gal. Oct. 1. What does the store pay for water for September, if the rate is 25 ¢ per thousand for the first 10,000 gal., and 20 ¢ per thousand for the second 10,000 gal. ?

3. A grocery company owns a delivery auto costing $ 1850. The monthly bill for gasoline and other expenses is $ 75 and for delivery man $ 60. The store averages 225 deliveries per day for a month of 26 da. Can the store afford to have its deliveries made by a central company at 3 ¢ *each* ? What will it lose or save in a year ?

213. Advertising.—The successful business man of to-day pays a great deal of attention to advertising. Many establishments employ an advertising manager at a good salary who gives it all of his time. Large stores even employ men who do nothing but trim windows. It will be impossible here to do more than mention advertising, an important phase of the selling expense.

214. Newspapers and Magazines.—Newspapers contain advertisements from local merchants, as well as from establishments of various sorts from outside the city in which the paper is printed. The "space" is paid for at so much per line or inch of each column. The price charged varies according to the circulation of the paper. How can the circulation affect the price a paper charges for its advertising space ?

Magazine advertising differs only in that they have no local advertisers. The price for space is also very much higher. Why ? For instance, a page in a magazine ranges from $ 100 to $ 3000 or $ 4000 per month. Cover and colored pages are, of course, even higher.

In order to find which magazine produces the best results, many advertisers keep a strict account of all letters resulting from their various magazine advertisements. In one magazine they may ask you to write for their catalogue m, in another for catalogue n, and so on. How can they tell from this in which magazine you saw their advertisement ? Their record shows not only the number of inquiries but also sales resulting from advertisements in each magazine. What is the purpose of this ?

Large business firms engage professional writers and artists to make up their advertisements. How can they afford this ? Mention some well-known advertisements.

EXERCISES

1. An advertiser in a local paper pays $ 1.75 per day for a month of 26 da. Because of carrying the same space for a month his bill is reduced ⅕. How much will this be per month ? per year ?

2. A manufacturing establishment advertised in five magazines, A, B, C, D, and E. Their rates per month were $ 650, $ 425, $ 250, $ 250, and $ 125. How much was this for magazine advertising per month ? per year ? Who would ultimately pay for the advertising ?

3. The establishment in Ex. 2 kept a checking system and found that the following number of letters were received: A, 369; B, 348; C, 167; D, 132; E, 73. What was paid for each inquiry ? If the firm decided to advertise in the three magazines that produced the most inquiries, which three would this be ?

4. The checking system further showed that the following number of sales resulted from these advertisements: A, 247; B, 196; C, 124; D, 107; E, 44. What was the advertising cost of each sale ? If the advertising was to be continued in the three magazines with the lowest cost of sales, which three would this be ?

5. How many sales were made altogether ? What was the advertising cost per sale ? How much must each sale be above cost plus all other expenses to pay for the advertising ?

6. How many sales were made from the three magazines with the lowest cost of sales ? How much was this per sale ?

7. Answer the questions in Exs. 2 and 5 by the use of *literal* numbers in the place of Arabic numbers.

215. **Other Forms of Advertising.**—The most common are:

Circulars sent through the mails or by carriers.

Samples distributed through the mail or by carriers.

Posters on bill-boards vary in expense according to size, time on the boards, location in the city, and size of the city. Show how all these items enter into the value of a billboard for advertising purposes.

Street-car sign space is sold by the month. The price varies with the size of the city and the number of cars containing the signs. Show how these items affect the value of street-car advertising.

One of the best advertising mediums of any establishment is **good service** and **reliability.** · Explain.

EXERCISES

1. A new establishment sent a circular to each family in a city of 15,000. If the cost of printing and mailing was 4 ¢ per circular, what was the cost, estimating 5 persons to each family?

2. Through the carelessness of a cheap clerk a store lost a customer whose account averaged $ 30 per month. How much business did they lose on this customer for the year?

3. Posters are made by putting together sheets 28 in. by 42 in. What height would be required to take 3 sheets the short way up and down?

4. A small city operates 34 street-cars. What will be the cost of a card in each car for 6 mo. at 40 ¢ per car each month?

5. A firm had 16 posters, each containing 12 sheets, placed in a city. What will be the total cost for a month at 15 ¢ per sheet?

216. Depreciation.—A shoe factory installs a machine costing $ 1800, which it is estimated can be used 10 yr. and then sold for $\frac{1}{5}$ of its cost. Should the depreciation of this machine be counted as a part of the expense of the last year it was used, or should a part of the depreciation be added to each year's expenses? Such forms of deprecia-, tion are very common.

217. Goods out of Season.—Goods out of season and sometimes goods which have been sold and returned are sold at cost or even below.

218. Bad Debts.—Many merchants yearly lose considerable sums of money from debts which are not paid. How can this form of expense be greatly reduced?

219. Waste.—Some of the above-mentioned expenses are really waste. Mention some. Other forms of waste arise all the time and in all places. Fruit spoils in the grocery; a poor meat-cutter wastes meat in the market; mice may greatly damage goods in stores; the coal-driver handles coal carelessly, thereby spilling a large quantity; a nail driven into a box may damage a costly piece of furniture; poorly packed dishes will be broken in transit; the government estimates that 14 % of the food coming into the average home is wasted; Jane does not study in school and so must study after school when she ought to get exercise at play; James gets careless when washing windows and breaks a pane. Mention other forms of waste.

220. Decreasing Selling Expense.—Selling expenses can be decreased by increasing the sales of each salesman. How?

EXERCISES

1. What is the estimate of the value of the machine in Art. 216 at the end of the 10 yr. ?

2. What is the depreciation in the 10 yr. ? What is this per year ? Make a graph, showing yearly value of machine. Show yearly depreciation from same graph.

3. A merchant bought a delivery automobile for $ 1850. He estimated, he could use it 6 yr. and then be allowed $ 525 for it in buying a new machine. What is the estimated yearly depreciation of the auto ? Make a graph, as for Ex. 2.

4. A merchant found that his sales one year amounted to $ 56,800 and that he lost on bad debts $ 495. What per cent of the total sales did he lose through bad debts ? Who paid this expense ?

5. According to the average found by the government (see Art. 219), what is the value of the food wasted·per month by the family whose monthly bill is $ 25 ? $ 65 ? What is the waste per year in each case ? What is the waste per year if the monthly bill is $ D ?

6. If it cost James $ 1.40 to pay for a window which he broke and if he earns 25 ¢ per hour while working, how many hours must he work to pay for his carelessness ?

7. Electric light in a certain city costs 10 ¢ per KW or 1000 watts. How much does George waste by forgetting to turn out three 25-watt lamps in the basement at night, if they burn 8 hours ? What will it amount to if he forgets the lights 14 times during the winter ?

8. Suggest some form of waste that you know about and make up a problem in numbers about it. Solve the problem.

9. Two salesmen in the same department of a store receive $ 14 and $ 22 per week each. The former sells $ 109 while the latter sells $ 214 worth of goods during the week. Which salesman is the more economical for the store ?

10. When business falls off, the clerks paid the lowest wages are the first to be discharged. Is this economy ?

PROFIT AND LOSS

221. Business Motives.—The first business principle is to buy at one price and sell at a higher price. Often the commodities bought are changed very much into others that are sold. For instance, the shoe manufacturer buys leather, nails, thread, blacking, and so on, and in addition he buys labor. All these he turns into shoes, which he sells. The jobbers and large wholesalers buy the shoes put out by the manufacturer. The jobbers sell to the small wholesalers and to the large retail stores. Wholesalers sell to retailers, who in turn sell to the consumers.

222. Net and Gross Costs.—The price that the retail dealer pays for the shoes, mentioned above, is called the **prime** or **net cost**. To this is added freight, drayage, and any other expenses (buying expenses) necessary to placing the shoes upon the shelves ready to sell, which is called **gross cost.**

As an equation this is:

$$GC = NC + B_{exp}. \qquad (1)$$

223. Gross and Net Profits.—**Gross profit** equals sales less gross cost. From this must be deducted all selling expenses, as was discussed in the last chapter. In equations these become:

$$GP = S - GC \qquad (2)$$
$$NP = S - GC - S_{exp} \qquad (3)$$
$$= S - (GC + S_{exp}). \qquad (4)$$

Net profit equals selling price less net cost and total expenses, or

$$NP = S - (NC + T_{exp}). \qquad (5)$$

EXERCISES

State what is bought and what is sold by the men conducting the following establishments:

1. Bakery. 3. Meat-market. 5. Sawmill.
2. Grocery. 4. Bank. 6. Shoe-repair shop.

State as far as possible what is bought and what is sold by the following:

7. Dentist. 9. Teacher. 11. Carpenter.
8. Farmer. 10. Lawyer. 12. Factory foreman.

13. A merchant bought goods for $ 1650 which he sold for $ 2364.50. Find the gross profit. Find the net profit if his expenses were $ 219.75.

14. A house was bought for $ 2650 and $ 328 spent in repairing it. It was later sold for $ 3400. Find the gross cost and the net profit.

15. A merchant sold goods for $ 1267, for which he paid $ 867. What were his gross and his net profits if his total expenses were $ 187 ?

16. What is the gross yearly income from a house that rents for $ 45 per month ? If the yearly expenses are $ 167.50, what is the net yearly income ?

17. Explain fully equation (1) on the preceding page. Solve it for net cost; for buying expenses.

18. Explain fully equation (2). Solve it for sales; for gross cost price.

19. A dry-goods merchant bought a bill of goods at $ 32.50, upon which he estimated a gross profit of $ 6.50. Find the selling price.

20. The gross cost of a bill of goods was $ 1965 and the selling price $ 2645. Find the gross profit. If the selling expense was $ 243, what was the net profit ?

21. Explain fully equation (3) on page 178. How is equation (4) obtained from (3)? Solve (3) for sales; for gross cost price; for selling expense.

22. If goods are sold at a loss, what is the relation between selling price, selling expense, and gross cost? What sort of a number will NP be in equations (3) and (4)?

23. A merchant bought goods for $ 1975 and sold them for $ 2132.50. Find the net gain if his expenses in selling the goods were $ 219.

24. Find the net profit, called **operating revenue,** for each railroad according to the following statistics for 6 mo.:

R. R. systems	Gross income	Total expenses
C. B. & Q.	$ 62,097,596	$ 54,791,299
Erie	36,088,083	31,100,040
Southern Pacific	71,202,273	58,444,609
Great Northern	39,057,427	38,475,495
Northern Pacific	42,023,234	33,936,893

25. Read the above numbers to the nearest million. Operating revenue is what per cent of gross income for each road? Arrange the roads in order according to this per cent.

26. A grocer charges 25 ¢ per can or $ 2.75 per dozen for peas. How much is saved by buying 3 doz. at one time? How many dozen must be bought to save $ 3? to save $ 1.50? to save $ 4.50?

27. How many cans must be bought at the rate given in Ex. 26 so as to save 1 doz. cans at the price of a single can?

28. Which is better and how much—to pay $ 1.75 per yard for dress goods or $ 1.45 per yard for a remnant of $7\frac{1}{2}$ yd. if only 6 yd. are needed, providing the quality is the same?

29. Would you rather receive $ 5.40 per week or $ 20.80 *per month?*

224. Selling Expense.—It was found in the last chapter that the greatest expense is that designated as selling expense. One year a grocery store sold goods for $ 64,000 which cost $ 49,500 when placed in the store. The net profits were $ 6100. From equation (3), page 178, find the total selling expense. Unless price of labor and other items change, the selling expenses for the next year can be estimated to be the same per cent of the gross cost as they were for the present year. Stated as an equation this is:

$$S_{exp} = GC \times \%S_{exp}. \qquad (1)$$

Hence, equations (3) and (4), page 178, become:

$$NP = S - GC - GC \times \%S_{exp} \quad (2) \text{ Explain.}$$
$$= S - GC(1 + \%S_{exp}). \qquad (3) \text{ Explain.}$$

EXERCISES

1. It costs a department store 18 % of the cost price of a certain class of articles to sell them. Find the total selling expense of articles whose gross cost price is 90 ¢; $ 2.40; $ 16.50; $ 35.

2. What will be the selling expense, which is 15 % of the gross cost, of an article whose gross cost is $ 5.46 ? $ 2.35 ? 96 ¢ ? $ 45.75 ? For what must these articles be sold before any profit will result ?

3. A dry-goods store one year sold goods whose gross buying price was $ 46,500, sales $ 63,700, and net profit $ 4900. Find the total sales expense. Find the per cent of sales expense.

4. Solve equation (1) above for per cent of selling expense.

5. Find the per cent of selling expense of the grocery store mentioned in Art. 224.

225. Per Cent of Profit or Loss.—One merchant invested $ 1200 and made a net profit of $ 200, while another merchant invested $ 400 and made a net profit of $ 80. Which made the better investment ? While a gain of $ 200 is more than a gain of $ 80 this difference alone does not show which is the better investment. Suppose that the second merchant had invested $ 1200 at the same rate of gain as he invested the $ 400, what would have been his gain ? The first merchant gained only $16\frac{2}{3}$ % of the amount invested, while the second merchant gained 20 % of the amount invested.

Hence, to compare the profitableness of investments we find the per cent of profit. This per cent of profit is variously computed. In the above we found the per cent of the investment that was earned. Merchants often find what per cent the gain is of the selling price. The per cent of profit may be computed either upon the buying price or upon the selling price, but whichever is used must be stated definitely. To say that 15 % was gained is ambiguous. Explain.

226. Equations of Per Cent.—If the net profit is found as a per cent of the gross cost, or investment, then

$$NP = GC \times \%P. \qquad\qquad (1)$$

Here $GC = NC + B_{exp}.$ (2) Explain.

Also $S = GC + NP + S_{exp}.$ (3) Explain.

From (1) and (3)

$$S = GC + GC \times \%P + S_{exp}, \qquad (4)\ \text{How ?}$$
$$= GC(1 + \%P) + S_{exp}. \qquad (5)\ \text{How ?}$$

If selling expense is given as a per cent of gross cost,

$$S_{exp} = GC \times \%S_{exp}, \qquad\qquad (6)$$
$$S = GC + GC \times \%P + GC \times \%S_{exp}, \quad (7)$$
$$= GC(1 + \%P + \%S_{exp}). \qquad\qquad (8)$$

If the per cent of profit is computed upon the selling price, then the equations will be:

$$NP = S \times \%P. \qquad (9)$$

Hence, from (3) and (9)

$$S = GC + S \times \%P + S_{exp}, \quad (10) \text{ Explain.}$$
or $\quad S - S \times \%P = GC + S_{exp}, \qquad (11) \text{ How ?}$
and $\quad S(1 - \%P) = GC + S_{exp}, \qquad (12) \text{ How ?}$

hence, $\qquad S = \dfrac{GC + S_{exp}}{1 - \%P}. \qquad (13) \text{ Explain.}$

EXERCISES

1. Goods are sold so as to realize a net profit of 20 % on the purchase price of $ 540. What is the net profit ?

2. A dry-goods merchant bought a bill of goods for $ 1350. Find gross cost if buying expense was 10 % of the bill.

3. The selling expense is estimated at 18 % of gross cost and it is expected to make 15 % profit. The selling price is then what per cent of gross cost ? Find the selling price of goods bought at a gross cost of $ 2350.

4. A clothing-store company bought goods for $ 1760, including all buying expenses. It is estimated that it will cost 17 % of the gross cost to sell the goods. For what per cent above cost must the goods be sold in order to realize a net profit of 20 % on the gross cost ? How much is this ?

5. What will be the selling expense of the goods sold in Ex. 4 ? What will be the gain on the goods sold in Ex. 4 ? What will be the selling price ?

6. Mr. Winslow bought goods for $ 1986 including all buying expenses. He estimates that it will cost him 17 % of the gross buying price to sell the goods. For what must he sell them to gain 15 % on the gross buying price ?

7. An automobile bought for $ 960 was sold at an advance of $16\frac{2}{3}$ %. What was the selling price ?

8. A house was purchased for $ 3200 and $ 270 spent in repairing it. Find the per cent of gain on the investment if sold for $ 3800 ? for $ 4200 ? for $ 3300 ?

9. An article bought for $ 160 is sold for $ 200. What is the gain and the gain per cent on the cost ?

10. Solve equation (1), page 182, for %P.

11. A bicycle bought for $ 35 was sold at $ 42. What was the per cent of profit on the cost price ?

12. A bill of dry goods amounted to $ 340 including all buying expenses. The selling price was $ 420. What was the gross gain and the gross per cent of gain on the cost ?

13. Find the net gain and the net per cent of gain on the goods of Ex. 12 if it requires 13 % of the cost to sell them.

14. An auctioneer bought a bankrupt stock of goods for $ 2300, which he sold for $ 3645.25. If his expenses were $ 687, what was his net profit ? What per cent of profit on the investment did he make ?

15. What per cent of profit was made on the selling price of the goods in Ex. 14 ?

16. Was the per cent of gain on the cost or on the selling price the higher ? Will this always be true ? Prove this. Hence, does it make a merchant's profit seem less by computing the per cent on the selling price ?

17. Solve equation (5) on page 182 for gross cost. If S_{exp} is omitted, what does the equation become ?

18. In order to sell an article for $ 2.50 and gain 25 % on the total cost, it must be bought for what ?

19. A real-estate broker sells a house for $ 2700. For *what* must he buy it to gain 5 % on the purchasing price ?

20. For what must berries be bought by the crate to sell for $ 3.50, in order to gain 20 % on the gross cost ?

21. The total cost of an article is $ 10. What should the selling price be in order to gain 20 % on the total cost ? What will be the gain ? The gain will be what per cent of selling price ?

22. A merchant has his goods marked so that he gains $16\frac{2}{3}$ % of the selling price. What does he gain on articles sold for $ 6 ? for $ 10 ? for $ 1.50 ? for 45 ¢ ?

23. Solve equation (9) on page 183 for S; for $\%P$.

24. Solve equation (10), page 183, for $\%P$. What does this become if S_{exp} is omitted ?

25. The total cost to a dealer of a bicycle is $ 24. If he sells it for $ 32, what per cent of the selling price does he gain ? What per cent of the total cost does he gain ?

26. A dealer bought fishing-tackle so that the cost and the total expenses were $ 1.65 each. If he sold it for $ 2.35, what was his gain per cent on the selling price ? What was his gain per cent on the total cost ?

27. Solve equation (10), page 183, for selling price. What . does this become if S_{exp} is omitted ?

28. The total cost of chicken-wire netting to a hardware dealer was $ 6.20 per roll. For what must he sell it in order to gain 20 % on the selling price ? What will he then gain on each roll of netting ?

29. For what must the wire in Ex. 28 be sold if selling expense is 15 % of gross cost ?

30. A sporting-goods dealer bought baseball gloves for $ 2.80 each. For what must he sell them to gain 25 % on the selling price ? What will he then gain on each ?

31. For what must baseball suits that cost $ 14 be sold in order to gain 15 % on the selling price ? 18 % ?

32. Make a graph showing a gain of 20 %.

227. Thrift.—Profit and loss is a subject for study not only by the merchant and the manufacturer but by all. Thrift requires a constant watchfulness over income and expenditure. It means the elimination of waste in every manner possible. Many persons waste by buying foolishly. How ? · Many try to save by buying a cheaper article which causes an added expense later. Give an illustration. Others decrease their income because they do not make the most of their earning capacity. Give an example.

EXERCISES

1. Mary considers buying a gingham dress for $ 12.50. She finds, however, that she can get a better dress at less cost by buying 5 yd. material at 49 ¢ per yard, 1½ doz. buttons at 50 ¢ per dozen, ⅓ yd. linen for a collar at $ 1.50 per yard, and 25 ¢ for thread and a pattern. How much does she save ?

2. Last year Mr. Wortman used 12 T. of coal which he bought at $ 6.40 per ton. Two boys have applied to take care of his furnace during the year—John at $ 1.25 per week and Horace at $ 1 per week. John is careful and uses 10 % less coal than Horace. Which will it be the more economical to employ and how much, if the boys are employed 24 wk. ?

3. Richard found that he could buy the seeds for his garden from one firm for $ 1.80, while he would have to pay $ 2.40 to another firm. What per cent of the cheaper seeds did he save by buying these ?

4. Richard sold for $ 16.50 the vegetables he raised from the garden that he planted from the cheaper seeds. If 40 % of the seeds were not fertile, what was his loss at the the same rate that he sold the vegetables he did raise ?

5. If the more expensive seeds had been only 15 % not fertile, what would have been the loss ? Compare the two *classes* of seeds.

6. Supply literal numbers in Exs. 4 and 5, and state Richard's loss by an equation.

7. The Homer and Buryns law firm hires at $12 per week a stenographer who makes so many errors that she must do $\frac{1}{4}$ of her work over. What could the firm afford to pay her if she did not make these mistakes?

8. Mr. Jelks has a position that pays him $200 per month. He desires to enter business for himself. As a prospective purchaser of a store he is told that he can gain an average of 15 % of the selling price of all goods sold. To what must his monthly and yearly sales amount in order that he may still have a net income equal to his wages?

9. Upon taking over the business Mr. Jelks found that he could make a profit of only $12\frac{1}{2}$ % of the sales. If his sales were $11,800 for the year, what was his net income? How much is this below a salary of $200 per month? What did it cost him to be "his own boss"?

10. It has been estimated on good authority that 95 % of the business men in the United States are failures. State this by an equation. At this rate how many business men are actually successful in a city of 50 business men? 120 business men? 74 business men?

11. Mrs. Jacks found that she had to pay a maid $8 per week and board estimated at $4.50 per week. She also estimated that the maid wasted 20 % of their food bill of $65 per month. How much would the maid cost in all per week? per month of 4 wk.? per year of 52 wk.?

12. Mrs. Jacks decided to employ help at the rate of 40 ¢ per hour for 3 half-days—4 hr. each time—per week. What did she pay for this help per week? per month? per year? What was the saving per year over a maid, as stated in Ex. 11?

XVI

BILLS AND INVOICES

228. Bills and Invoices.—Statements of goods bought for future sales are called either **bills** or **invoices.** Such are statements of lumber bought by a lumber dealer or vegetables bought by a grocer. Suggest three other invoices. Statements of goods or services which are to be or have been used are called **bills.** We thus have a bill of lumber for a house or the janitor's bill for services. Suggest three other bills. The mark # before a number means "No." and after a number means pounds.

Bills and invoices should contain date of purchase; names of the one by whom and the one from whom purchase was made; articles and amount; price of each unit, as pound or dozen; price of amount of each kind; total amount of purchase; terms.

OLDMAN DRY GOODS CO.

Philadelphia, Pa., July 26, 1919.

IN ACCOUNT WITH

Mr. Thos. U. Kitchen,

Zora, Ark.

Terms: Net 30 days.

2 doz.	C thread No. 100	40 ¢	$	.80
6 "	C " " 40	40		2.40
8 "	C " " 50	40		3.20
3 "	B silk thread assorted 80			2.40
		Total	$	8.80

All bills and debts should be paid promptly when due. This is necessary in order to keep one's **"credit good."** File for reference all receipted bills paid with cash. Why? It is also a good practice to file **large** receipted bills and those which are payment **in full to some date**, whether paid by cash or check.

EXERCISES

1. Tell from the bill on the opposite page: who bought the goods; from whom the goods were bought; residence of the one selling the goods; residence of the purchaser; articles bought; total amount; when due.

Make out bills for the following, similar to the bill on the opposite page, as if the purchaser lived in your city:

2. Harry James bought of the Cross Dry Goods Co., New York City, Nov. 12, 1918: 12 yd. long cloth at 15 ¢ per yard; 51^2 yd. bleached muslin at $8\frac{1}{3}$ ¢ per yard; 53^3 yd. brown muslin at $4\frac{3}{4}$ ¢ per yard; 41^2 yd. outing flannel at $6\frac{1}{4}$ ¢ per yard; 4 comforts at $2.50 each; 5 pairs wool blankets #3K at $4.25 each; 3 pairs woollen blankets #6K at $6.65 each. Terms, cash in 30 days.

3. Henry Jordan bought from the Twin City Mill and Lumber Co., of Minneapolis, Minn., Mar. 27, 1918: 500 1″ x 12″ x 14′ yellow pine No. 1 at $37 per M; 300–1″ x 4″ x 16′ yellow pine No. 1 at $37; 600–2″ x 6″ x 16′ yellow pine No. 1 at $38.50; 2000 ft. cypress flooring at $52.50. Terms, net 30 days.

4. G. J. Gillmer bought of Stout Stationery Co., Chicago, Ill., May 22, 1918: 6 doz. #9 tablets at 48 ¢ per dozen; 50 reams typewriting paper at 75 ¢ per ream; 2 gross #17 pencils at $4.80 per gross; 3 doz. boxes fancy stationery #37A at $6.70 per dozen; 2 doz. boxes fancy stationery #15B at $8.85 per dozen. Terms, cash.

5. Rockcastle and Ranyon bought from Large Rapids Furniture Co., Grand Rapids, Mich.: 4 doz. dining-chairs at $ 56 per doz; 6 dining-tables at $ 32 each; 6 rockers # 7 at $ 14.50 each; 6 rockers # 14B at $ 9.75 each. Terms, 60 days.

6. Harry McClow bought from Reader Modock Grocery Co., Chicago, Ill.: 50 # coffee Sparkling Brand at 18 ¢; 200 # beans at $ 3.75 per C; 6 doz. Rise Brand baking powder at $ 2.80 per dozen; 4 gross Blush Brand canned peaches at $ 3.90 per dozen. Terms, 30 days.

7. Make up a grocery bill having at least six items, with yourself as purchaser and some city firm as seller. Use prices that are now common.

To DENVER ELECTRIC UTILITIES CO., Dr. *Bring This Card with You.*				Book No. 7 Cust. No. 314		
Present Reading 256 KW						
Former Reading 229 KW						
Amount Consumed 27 KW						
LIGHTING RATE: Discount, 10 %	Amount	2	70			
0 to 500 KW, 10c. Excess of 500 KW, 8c.	Discount		27			
POWER RATE: Discount, 10 %						
First 100 KW, .08. Next 100 KW, .07. Next 1300 KW, .06. Over 1500 KW, .05.	Net	2	43	Total		

On each electric-light bulb is stated the number of watt-hours of electric current it consumes per hour. A KW is 1000 watt-hours. If an appliance, as an iron or toaster, gives its amperage, A, multiply this by the voltage of the city *current,* usually 110, to get the watt-hours it burns per hour.
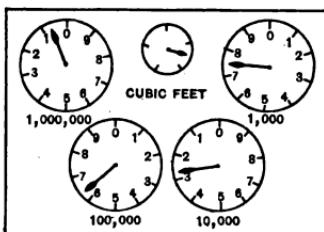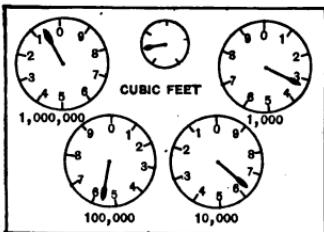
To **DETROIT GAS CO.,** Dr.									
PHONE 286			519 Merchant Street						
Reading taken Jan.......1919	3	5	2	3	0	0		2	80
Reading taken Dec......1918	3	4	6	8	0	0		3	75
Gas Consumed			5	5	0	0			55
							Cu. ft. at Sched. Rate $	8	10
							Discount......... $		81
Schedule of Gas Rates							*Net* .... $	7	29
Gross, $1.40 per M for first 2,000 cu. ft.							*Gas Arrears*....... $		
Gross, $1.25 per M for next 3,000 cu. ft.							*Sundry Items*.... $		
Gross, $1.10 per M for next 5,000 cu. ft.									
Gross, $1.00 per M for next 10,000 cu. ft.							*Total* .... $	7	29
Gross, $ .90 per M above 20,000 cu. ft.									

In gas and water meters the dial to the right reads hundreds, the next reads thousands, the next reads tens of thousands, and so on. Electric meters read to the nearest KW. What does each of the above gas meters read ? Find the difference between their readings.

EXERCISES

1. Make out an electric bill for a meter reading Oct. 30, 654 KW and Nov. 30, 672 KW, at 11 ¢ per KW.

2. Read the electric meter at your school or your home now and a week later. Make out a bill for the current used.

3. Check the above gas bill.

4. Make out a bill for the gas used one month if the reading at the beginning of the month was 32,700 cu. ft. and at the end was 36,200 cu. ft., at $ 1.15 per 1000 cu. ft.

5. Repeat Ex. 2 for a gas meter.

```
CITY WATER              January 1, 1919.
St. Paul, Minn.         Account for December, 1918.

State of Meter........235600  Street   State
Former Reading........219800  Number   1119
Consumed............. 15800   Total bill..........$3.66
Rate per 1,000 gallons:
          First 10,000.... .25   Next 25,000.... .15
          Next 10,000..... .20   Next 35,000.... .14
          Next 10,000.... .18    Next 50,000.... 13
          Next 10,000.... .16    Next 50,000.... .12
```

EXERCISES

1. Check the above bill.

2. The reading of a water meter was 240,500 gal. July 28 and 245,100 gal. Aug. 30. Make out a bill for the water used at the rate of 25 ¢ per 1000 gal.

3. Read the water meter at your school or your home now and a week later. Make out a bill for this at the rate charged for water in your city.

4. A certain city charges 30 ¢ per 1000 gal. for the first 10,000 gal. and 20 ¢ per 1000 gal. for the next 10,000 gal. used. Make out a bill for June if the reading May 29 was 450,800 gal. and June 30, 467,300 gal.

5. Make out an electric bill for a meter reading Aug. 29, 2370 KW and Sept. 28, 2440 KW, at 12 ¢ per KW.

6. Express by an equation the cost of running n electric lights each marked w watts for h hours at c cents per KW.

7. Use the equation found in Ex. 6 to find the cost of running some lights you know about for a given time.

229. Monthly Statements.—It is the custom of most establishments which have credit customers to issue to them *monthly* statements of their accounts. These statements

```
┌──────────────────────────────────────────────────────┐
│          HENRY M. ROWE HARDWARE COMPANY               │
│                  Minneola, Tenn., June 1, 1919.       │
│  IN ACCOUNT WITH                                      │
│                    Mr. Henry G. Weeks.                │
│  ════════════════════════════════════════════════════│
│  May 1  to bal.  - - - - $7.50                        │
│  May 4  to mdse. - - - -  .90                         │
│  May 15 to  "    - - - - 5.40                         │
│  May 21 to  "    - - - - 8.30                         │
│  May 27 to  "    - - - - 2.15 - - - $24.45            │
│  May 21 by cash  - - - - 12.00 - - -  12.00           │
│                               Bal.  $12.45            │
└──────────────────────────────────────────────────────┘
```

vary from the one giving merely the total amount due to the one stating each item bought during the month, as well as any balance due from the previous month. An illustration is found above. Check this statement.

<div align="center">

EXERCISES

</div>

1. Make out a statement to Henry Works from the Jerremy Grocery Co.: Balance due, $ 5.34; July 2, $ 6.75; July 7, $ 11.45; July 11, $ 7.35; July 14, $ 6.72; July 19, $ 10.68; July 23, $ 3.56; July 28, $ 14.85.

2. Make a statement to Geo. R. Beeman from the Jenks Lumber Co.: Balance, $ 145.35; Mar. 5, $ 45.60; Mar. 12, $ 104.45; Mar. 23, $ 73.25.

3. Make a statement to Henry Wilson from the St. Louis News for papers furnished him during May: week of May 3, /150; May 10, 150; May 17, 150; May 24, 150. Cost of each copy was 9 ¢ per week. Give the weekly and total amount for the month.

4. Make up a statement, using your name and that of some firm in your city. Give at least four items.

XVII

COMMERCIAL DISCOUNTS

230. Reasons for Discounts.—Commercial or trade discounts are reductions from original prices. These arise frequently. The retailer makes a reduction on damaged articles, goods sold out of season, and on whatever he may have an oversupply. The wholesaler sells at a discount odd lots, an oversupply, especially large orders, and for prompt payment. The bill at the top of page 197 states that it is due in 60 da., but that 2 % discount will be given if paid within 30 da., and 4 % if paid cash, that is, within 10 da.

Wholesale firms issue expensive catalogues which it would be exceedingly costly to reissue with every change in price. Furthermore, a retail merchant may often make a sale of goods not in stock by showing a catalogue to his customer. If the wholesale prices were printed in the catalogues this could not be done. Hence, it is customary to print in catalogues a price higher than the actual price and then provide the retail merchant with a private discount list. This discount list can be renewed cheaply and simply as prices change.

231. Forms of Discounts.—In retail trade discounts are often a stated amount or a stated new price. In wholesale trade discounts are as a rule a per cent of the original marked price. State an illustration.

232. Finding the Discount.—The discount, D, is always a certain rate or per cent, $R\%$, of the marked price, M_p. Hence,

$$D = M_p \times R\%. \qquad (1)$$

233. Finding the Net Price.—The net price, N_p, is what remains after subtracting the discount from the marked price. Thus,

$$N_p = M_p - D. \qquad (2)$$

By replacing D with the value found in (1) we get

$$N_p = M_p - M_p \times R\%, \qquad (3)$$

then $\qquad N_p = M_p(1 - R\%). \qquad (4)$

Explain equation (4) fully.

EXERCISES

1. Find the discount upon goods valued at $450 if the rate of discount is 5%; $12\frac{1}{2}\%$; $33\frac{1}{3}\%$.

2. A bill of goods amounting to $845 can be discounted at 3% if paid cash, but need not be paid for 2 mo. What will be the discount if it is paid cash?

3. If money is borrowed at the bank to pay the above bill, the interest rate being 8%, what will be saved? What will be the per cent of saving?

4. Mrs. Baker finds that canned goods which cost her $28\cancel{c}$ each can be bought by the dozen at a discount of 7%. What will she save on 5 doz. in buying them by the dozen?

5. Find how much the 5 doz. of canned goods cost Mrs. Baker by equation (2); by equation (4).

6. What per cent of the marked price is paid when the discount is 10%? 15%? $33\frac{1}{3}\%$? 43%? 28%? 63%?

7. What is the quantity inside the parentheses in equation (4)? Apply to Ex. 6 above.

```
┌─────────────────────────────────────────────────────────────┐
│              LOUIS C. KARPENTER CO.                         │
│                                                             │
│                    Cleveland, Ohio, May 17, 1919.          │
│  IN ACCOUNT WITH                                            │
│                    Mr. Henry R. Goode,                      │
│                                                             │
│                            Mayville, S. D.                  │
│  Terms· Cash ın 30 days.                                    │
│                                                             │
│  2 doz. Fancy peaches     $ 3.15 - -    $                   │
│  3 doz. Prime grape jelly   1.45 - -                        │
│  1 doz. Asparagus tips      3.65 - -                        │
│  2 doz. Choice apricots     2.95 - -                        │
│                                 Total $                     │
│              .              Less 25 %                       │
│                             Net         $                   │
└─────────────────────────────────────────────────────────────┘
```

8. Complete the above bill; find the gross cost and the net cost.

Make out bills like that above for the following:

9. Bought of K. R. Marks and Co., Trenton, N. J., by Heron Tripps (your city): 3 cases salmon at $ 7; 5 cases corn at $ 6.50; 2 cases peas at $ 8.50; 7 cases soup at $ 7.25. Discount, 15 %.

10. Bought of Superior Milling Co., Minneapolis, Minn., by K. W. Story (your city): 50 bbl. extra best flour at $ 14; 20 bbl. graham at $ 12.50; 10 bbl. whole wheat at $ 15.50; 50 small sacks buckwheat at 40 ¢ each. Discount, $16\frac{2}{3}$ %.

11. Bought of Western Text Book Co., Chicago, Ill. (use your own name and city) the following bill: 80 arithmetics at 65 ¢; 60 U. S. histories at 75 ¢; 40 elementary-science books at $ 1.10; 70 advanced geographies at 95 ¢. Discount, 20 %.

J. M. FISHER HARDWARE CO.

Chicago, Ill., Apr. 17, 1919.

IN ACCOUNT WITH

Mr. Harry V. Jasper,

Oldville, Ark.

Terms: 4% cash, 10 days; 2%, 30 days; net, 60 days.

1	doz.	M and K hoes	$ 8.45
2	doz.	G and R rakes	10.75
3	doz.	Superior garden tools	2.15
8		Lawn-mowers	3.65
			Total $

12. Complete the above bill and find for what it can be paid within 10 da.

Make out bills like the above for the following and compute the net cost.

13. Bought of H. J. Pape and Sons, New York, by (use your own name and city), the following bill: 16 pairs curtains at $8.50 per pair; 6 mats 9′ x 12′ at $18.50; 6 mats 8′4″ x 10′6′′ at $16.75; 8 drapes at $13.25. Discount will be 5 % cash; 3 %, 30 da.; 1 %, 60 da.; cash, 90 da.

14. Bought of A. Q. Rossiter, Chicago, Ill., by J. Z. Wise (your city): 4 dining-tables at $38; 3 doz. dining-chairs at $2.50 each; 4 rockers at $14; 3 lady's desks at $32. Discount, 3 %, 10 da.; 2 %, 30 da.; net, 60 da.

15. Suggest and make out a bill having a discount for some commodity sold in your town.

16. Solve equation (4), page 195, for M_p.

17. Solve equation (3), page 195, for R %.

234.   Finding the Marked Price.—Merchants sometimes desire to mark articles at a price so that they may offer a given discount and sell them at some assigned net price. Solving equation (4), page 195, gives

$$M_p = \frac{N_p}{1 - R\,\%}. \qquad \text{(1)   Explain fully.}$$

235.   Finding the Rate of Discount.—The wholesale merchant has a marked price in his catalogue on goods which he must sell at a given net price. He must then compute from the marked and selling price the discount rate to send to his customers. Solving equation (3), page 195, for $R\,\%$ gives

$$R\,\% = \frac{M_p - N_p}{M_p}. \qquad \text{(2)   Explain fully.}$$

EXERCISES

1.   For what must an article be marked so as to permit a discount of 20 % and still sell for $ 12 ?

2.   What must a suit of clothes be marked so as to permit a discount of $33\frac{1}{3}$ % and sell for $ 16 ?

3.   What must overcoats be marked to sell for $ 25 and still give a discount of $16\frac{2}{3}$ % ?

4.   Suits selling for $ 12 must be marked what to give a discount of 20 % ? of 25 % ? of 40 % ? of $33\frac{1}{3}$ % ?

5.   Canned corn sells at 24 ¢ per can or at 21 ¢ per can if bought by the dozen. What is the rate of discount when bought by the dozen ? What is saved per dozen ?

6.   What is the rate of discount in buying canned goods at $ 2.25 per dozen that cost 21 ¢ per can ?

7.   When pencils sell at 10 ¢ each or 3 for 25 ¢, what is the rate of discount ?

8.   If possible, bring in data on some "special sale" that *has been* held in your city. Compute some rates of discount.

9. A wholesale firm finds that it must change its prices due to an advance in cost of production. How can they acquaint their customers of the changes most cheaply ?

10. Tables marked $ 36 in a catalogue must be sold for $ 27. What will be the rate of discount ?

A wholesale firm gives its customers a discount of 20 % on all goods. What must be the catalogue prices of goods to sell at the following wholesale prices:

11.	$ 1.20	**13.**	$ 0.80	**15.**	$ 0.60	**17.**	$ 2.00
12.	$ 2.40	**14.**	$ 1.60	**16.**	$ 3.60	**18.**	$ 1.12

19. Find the retailers' profit on the goods listed in Exs. 11–18 if sold at catalogue price.

20. Dining-chairs listed at $ 7.50 in the catalogue must be sold at $ 5.25. Find the discount rate.

21. If the chairs in Ex. 20 have advanced $ 1.25 each, what will be the discount rate ?

22. In Ex. 20 the $ 1.25 is what per cent of the catalogue price of the chairs ? The per cent advance of the list price plus the new discount rate equals what ?

23. Desks listed in the catalogue at $ 32 have increased $ 4 in price. What is the decrease in discount rate ?

24. For what must a retail dealer sell the chairs in Ex. 20 to make a profit of 25 % on the cost ?

25. For what must a dealer sell the chairs in Ex. 20 to make a profit of 25 % on the selling price ?

26. Mr. Kohler's bills are due in 60 da., but he receives 4 % discount by paying cash. If he pays cash for his bills, he is receiving what rate of interest on the money thus invested ?

236. Discount Series.—It often happens that two or more discounts are given on a bill. For instance, a bill may give a discount of 25 % from the catalogue price and then 5 % discount from this price if paid cash. Payment within 10 da. is reckoned as cash. It is customary to say that the discount would be 25 % and 5 %. Several years ago a certain firm gave three discounts on all of its catalogue goods: 70 %, 10 %, and 5 %.

EXERCISES

1. The catalogue price of an article is $ 12. What is the net price if the discount is 25 % ? Suppose that a further discount of 5 % is given for cash payment, what will be the net price ?

2. An article listed at $ 15 in a catalogue is subject to three discounts of 30 %, 20 %, and 10 %. What will be the net price ?

3. Rewrite the bill at the top of page 196 making the terms 5 % cash; 2 %, 30 da.; net, 60 da. How much will it take to pay the bill within 10 da. ? 30 da. ? 60 da. after purchase ?

4. Discounts of 40 %, 30 %, and 10 % are given on certain goods in a catalogue. What will be the net price of goods listed at $ 8 ? at $ 12 ? at $ 15 ? at $ 7 ?

237. One Discount Equivalent to a Series.—It is often convenient to have one single discount in the place of several. The net price may be desired of each article in the bill for Ex. 3, or for each article in Ex. 4 above. It is much simpler to find the single rate of discount that is equivalent to the two or three given rates of discounts.

Let the discount series be R_1, R_2, and R_3. Suppose that *the net* price after the first discount has been deducted is

N_1, after the second discount has been deducted N_2, and after the final discount has been deducted the net price is N_f. Equation (4) on page 195 is

$$N_p = M_p(1 - R\,\%).$$

From this

$$N_1 = M_p(1 - R_1),\qquad\qquad (1)\quad \text{How?}$$

also

$$N_2 = N_1(1 - R_2)$$
$$= M_p(1 - R_2)(1 - R_1),\qquad (2)\quad \text{How?}$$

and

$$N_f = N_2(1 - R_3)\qquad\qquad (3)\quad \text{How?}$$
$$= M_p(1 - R_1)(1 - R_2)(1 - R_3). \ (4)\quad \text{How?}$$

The multiplier of M_p is 1, less the discount.   See equation (4), page 195.   Hence, $(1 - R_1)(1 - R_2)(1 - R_3)$ taken from 1 gives the single discount.   Or,

$$R = 1 - (1 - R_1)(1 - R_2)(1 - R_3). \qquad (5)\quad \text{How?}$$

Thus, if the discount rates are 10 %, 20 %, and 30 %, the single discount rate is

$$R = 1 - (1.00 - .10)(1.00 - .20)(1.00 - .30)$$
$$= 1 - .90 \times .80 \times .70$$
$$= 1 - .504 = .496 = 49.6\ \%.$$

EXERCISES

1.   Find the single discount equivalent to 25 % and 5 % for Ex. 1 on the opposite page.   Use this to find the net price.   Is this the same as found before?

2.   Find the single discount equivalent to the three discounts in Ex. 2 on the opposite page.

3.   Find the single discount equivalent to the three in Ex. 4 on the opposite page.

4.   Show that the order of the discounts in a series is immaterial.   Apply to Exs. 1 and 2 above.

5.   Carry out the multiplications in equation (4) above. Verify by substituting .10, .20, and .30 for the R's.

XVIII

ACCOUNTS

238. Personal Accounts.—Merchants keep accounts with their customers who do not make all purchases for cash. The book for such accounts is called a ledger. On the left side of the page are entered goods, money, or other valuables received by the one with whom the account is kept. This is called the **debit—Dr.**—side. On the right side are placed payments. This is called the **credit—Cr.**—side.

	Dr.	*Wm. A. Cremona*				*Cr.*		
Feb. 1	*Bal.*		$ 7	45	*Feb.* 7	*By cash*	$ 15	00
Feb. 4	*To gds.*		18	36	*Feb.* 10	" *labor*	5	75
Feb. 10	" "		6	75	*Feb.* 28	" *cash*	23	00
Feb. 18	" "		13	65				
Feb. 24	" "		2	36		*Bal. (red)*	4	82
			48	57			48	57
Mar. 1	*Bal.*		$ 4	82				

239. Balancing Accounts.—To balance an account find the difference between the debit and the credit sides and enter this in **red ink** on the **smaller side.** The sum of both sides—the same—is placed below each and two lines drawn beneath this. The balance is then entered in black on the side opposite to the red entry. Balances are made monthly or when needed.

EXERCISES

Enter and balance the following accounts:

1. Jeoffry Royal. Dr.: May 1, balance $ 15.40; May 4, garden tools $ 4.30; May 8, garden seeds $ 1.45; May 24, chicken wire $ 12.85; May 28, kitchen utensils $ 7.65. Cr.: May 4, strawberries $ 2.35; May 29, cash $ 25.00.

2. Jason Bayel. Dr.: Aug. 1, balance $ 234.75; Aug. 7, lumber $ 38.65; Aug. 18, cement $ 8.50; Aug. 24, lath $ 12.30. Cr.: Aug. 12, labor $ 15.60; Aug. 29, cash $ 245.

3. Henry Kobal. Dr.: Nov. 2, groceries $ 3.57; Nov. 13, groceries $ 6.23; Nov. 24, groceries $ 5.18; Nov. 28, groceries $ 3.67. Cr.: Nov. 13, sweet potatoes $ 7.50; Nov. 25, pumpkins $ 6.45.

4. Jasper T. Kobes. Dr.: Nov. 5, suit clothes $ 35.00; Nov. 9, hat $ 3.50; Nov. 19, pair shoes $ 6.00; Nov. 22, underwear $ 8.40; Nov. 26, tie 50 ¢; Nov. 29, gloves $ 2.50. Cr.: Nov. 6, labor $ 8.50; Nov. 14, delivering trunk 75 ¢; Nov. 29, cash $ 46.65.

5. Jackson K. Normley. Dr.: Mar. 3, prescription $ 1.35; Mar. 6, fertilizer $ 5.60; Mar. 17, soap $ 1.15; Mar. 22, stationery $ 2.25; Mar. 27, prescription $ 1.65. Cr.: Mar. 5, spading garden $ 3.60; Mar. 9, planting part garden $ 1.25; Mar. 21, cash $ 7.15.

6. Jas. A. Dorley. Dr.: Aug. 1, balance $ 14.56; Aug. 5, wall paper $ 8.50; Aug. 8, labor (hanging paper) $ 6.90; Aug. 18, white lead $ 6.70, oil $ 1.80, coloring 45 ¢. Cr.: Aug. 8, collecting bill $ 12.45; Aug. 24, law opinion $ 5.00; Aug. 26, drawing agreement $ 2.50.

7. Check the account on the opposite page very carefully and correct any error.

240.   Impersonal Accounts.—Russell raises chickens, and in order to tell whether they are profitable or not he keeps an account with them. He debits the account with all that he pays out for them, as feed, house, fence, and so on. The chickens receive this. He credits them with all returns in eggs and poultry · sold. The chickens give · Russell this. The account below shows items on both sides. Name other impersonal accounts which you think should be kept.

Check the following account very carefully:

	Chicken Account						
	Dr.				*Cr.*		
Mar. 15	*To 20 chicks 8c*	*$1*	*60*	*Sept. 15*	*By 3 fries 85c*	*$2*	*55*
Mar. 15	*To feed*		*50*	*Oct. 29*	" *6* " *90c*	*5*	*40*
Apr 4	" *30 chicks 8c*	*2*	*40*	*Nov. 23*	" *13* " *85c*	*11*	*05*
Apr. 11	" *feed*	*2*	*50*	*Jan. 15*	" *2 doz. eggs 32*		*64*
July 15	" *lumber*	*14*	*75*	*Feb. 1*	" *3* " " *30*		*90*
July 29	" *wire*	*8*	*50*	*Feb. 12*	" *5 doz. eggs 28*	*1*	*40*
Sept. 15	" *feed*	*3*	*25*	*Mar. 1*	" *7* " " *21*	*1*	*47*
Nov. 29	" "	*4*	*00*	*Mar. 15*	" *value chicken-house and yard*	*30*	*00*
	Balance (red)	*27*	*91*	*Mar. 15*	*To value 18 chickens*	*12*	*00*
		$65	*41*			*$65*	*41*
				Mar. 15	*Balance*	*$27*	*91*

EXERCISES

Make and balance the following accounts:

1.   Mary made a garden and paid $ 1.30 for tools, Apr. 15; 20 ¢ for onion sets, Apr. 20; 40 ¢ for tomato plants, May 20. She sold onions for 45 ¢, May 26; radishes for 65 ¢, May 28; lettuce for 35 ¢, June 1; beans and peas for $ 1.15, June 10; beans for 70 ¢, July 1; tomatoes for $ 1.05, *Sept.* 5.

2. Harry has been raising chickens and found that he had a debit balance, Oct. 31, of $ 32.45. He sold 24 lb. chickens at 26 ¢ per pound, Nov. 10; 27 lb. at 28 ¢ per pound, Nov. 23; 30 lb. at 24 ¢, Dec. 7; 8 doz. eggs at 28 ¢ per dozen, Nov. 10; 11 doz. at 30 ¢, Nov. 24; 8 doz. at 32 ¢, Dec. 9; 6 doz. at 35 ¢, Dec. 21. He paid for feed: $ 1.75, Nov. 10; $ 2.35, Nov. 27; $ 3.80, Dec. 18.

3. Make up items for an account on keeping chickens or a garden, enter each item, and balance the account.

4. Gather data for an account on gardening, chicken-raising, paper-carrying, or some other enterprise engaged in by some member of your class and make out the account.

5. Mr. Wasner owns a house, which he rents. He keeps an account of the receipts and expenses. Jan. 1, received $ 90 rent, Jan. 15, paid taxes $ 56; Apr. 1, rent $ 90; May 5, papering and painting $ 18.50; July 1, rent $ 90; Aug. 4, repairs $ 14.35; Oct. 1, rent $ 90; Dec. 31, interest on the investment of $ 4000 at 6 %. On which side is the balance ? Is it a paying investment ?

6. Mr. Josrus, who is a farmer, kept the following account with 80 A. that he planted to corn: labor (ploughing and planting) $ 400; seed corn $ 47; labor (harvesting the corn) $ 680; interest at 6 % on investment of 80 A. at $ 70 per acre; taxes $ 43; the 2950 bu. harvested sold at $ 1.32 per bushel.

7. What were Mr. Josrus' expenses per acre according to the data in Ex. 6 ?

8. According to the data of Ex. 6, what was Mr. Josrus' income per acre ?

9. What was Mr. Josrus' net income per acre ?

10. Consider that Mr. Josrus had half of the necessary expenses invested all of the time. How much would that make his total investment ? How much per acre ? What rate of interest does he make on the money invested ?

241. Cash Account.—It is a good plan to keep accounts of cash similar to that given just below. Money coming in is debited to "cash"—that is, something given to "cash." Money paid out is credited to "cash"—it is something given up by "cash." Do you keep an account of the cash you earn and receive together with your expenses? If not, learn how to do so through these lessons.

		Cash Account								
Sept.	*1*	*Bal.Cash on Hand*	*$4*	*65*	*Sept.*	*3*	*Books*		*$3*	*85*
"	*10*	*Working*		*50*	"	*9*	*Tablet*			*10*
"	*15*	*Carrying papers, 2 weeks*	*3*	*30*	"	*14*	*Tie*			*35*
"	*17*	*Working*		*60*	"	*17*	*Football game*			*25*
"	*29*	*Carrying papers, 2 weeks*	*3*	*30*	"	*24*	*Cap*			*85*
		.			"	*30*	*Church and S.S. for month*			*60*
					"	*30*	*Savings-Bank*		*4*	*00*
					"	*30*	*Balance* (red)		*2*	*35*
			$12	*35*					*$12*	*35*
Oct.	*1*	*Bal.Cash on Hand*	*$2*	*35*						

EXERCISES

Fill out and balance the following account:

1. Jason began May with a balance of $ 2.14. He earned 45 ¢, May 2; 65 ¢, May 9; 65 ¢, May 16; 30 ¢, May 23. He spent 15 ¢ tablet, May 4; 10 ¢ candy, May 6; $ 1.25 baseball, May 11; 10 ¢ candy, May 22; 15 ¢ baseball game, May 23; church and Sunday-school for the month 40 ¢.

2. Suggest items for an account like the one given above. Fill out the account and balance it.

242. Household Accounts.—The following is a simple *form* of monthly household account. At the end of each *month* the totals are entered on a separate page which are

totaled at the end of the year. Such household accounts show for what the monthly income was spent and gives data for improvements in this spending. Money not used any one month is called **reserve**. If more is spent than the income, that is a monthly **deficit** which must be taken from former reserves or made up by later reserves.

Date	Food	Fuel Light	Rent	Clothes	Recreation and self-improvement	Charity	Insurance	Miscellaneous	Deficit or reserve
3	3.45		32.50					.42	
5	4.82				2.00	.50		1.35	
8	3.15			5.85					
10	1.60					.25		2.09	
12	4.20				.80	.50		.78	
15	3.85								
18	5.17				1.40	1.00		.85	
21	3.27	5.85				.50		1.36	
23	1.89			17.00	.40				
26	4.28							.89	
28	1.97				2.65	1.00			
30	2.08	2.13					11.45	3.17	?
Totals	?	5.98	32.50	?	?	?	?	?	?

EXERCISES

1. Find the totals for each item in the above account. If the monthly income was $ 170, what were the savings?

Find the totals, together with savings or deficits, for the following household account:

2. Groceries and meat $ 31.55; fuel and light $ 3.25; recreation and self-improvement $ 6.25; clothes $ 4.50; rent $ 30; miscellaneous $ 7.15; charity $ 2.50; insurance $ 10.50; reserve? Income, $ 145 per month.

243.   Inventories.—Business establishments make at least once yearly an itemized list, called an invoice, of all their possessions and the value of each.   This is called **taking stock** or **taking an inventory.**

244.   Resources and Liabilities.—The **resources** of a man or of a firm are its possessions.   This includes the inventory of goods and the equipment on hand, together with all good notes and accounts payable to it.   These are called notes and accounts **receivable.   Liabilities** are the negative of possessions; they are what the person or the firm owes. Notes, bills, and accounts which the person or the firm owes are called notes, bills, and accounts **payable.**   Resources less liabilities give net present worth.   Most establishments issue a statement of their resources and liabilities from time to time to those interested.   Many of these statements, like the bank statement found on page 103, are published in the papers.   Study carefully the bank statement on page 103 and the statement given below.   Complete the statement below.

Resources Mercer Dry Goods Co.					
· Resources					
Cash	On Hand	635	48		
Merchandise	Inventory	7854	75		
Real estate	Building	4500	00		
Fixtures		850	00		
Accounts receivable	Ledger	1359	67	????	??
Liabilities					
Accounts payable	Ledger	2089	76		
Bills payable	Note to bank and interest	706	30	????	??
Net capital				????	??

EXERCISES

1. Make a statement of resources and liabilities for the following conditions on Jan. 1, 1919: Invoice of goods on hand $ 1275; cash on hand $ 107.64; accounts receivable $ 678.34; Liberty Bonds $ 400; bills payable $ 456.08; note payable $ 600; interest on this note due for 6 mo. at 8 %.

2. Make a statement of resources and liabilities for the Roskay Dry Goods firm according to the following items: Inventory of equipment $ 1500; inventory of goods on hand $ 2350.67; cash on hand $ 415.74; Liberty Bonds $ 1500; accounts receivable $ 678.34; notes receivable $ 845; total interest due on bonds and notes $ 74.58; bills payable $ 2845.67; notes payable and interest $ 678.54.

3. Jan. 1, 1919, the books of M. V. Koleys show the following for which he desires you to make out for him a statement of resources and liabilities, showing his present worth: Inventory $ 2342.48; Liberty Bonds $ 900; cash on hand $ 219.67; accounts receivable $ 456.73; notes receivable $ 850; interest due on notes receivable $ 42; bills payable $ 947.69; note payable $ 1200; interest on note payable $ 72.

4. Make out a resources and liability statement for a farmer, Jan. 1, 1919: Owns farm of 240 A. worth $ 85 per acre; stock valued at $ 1650; grain valued at $ 1250; implements valued at $ 625; household goods valued at $ 400; cash in the bank $ 34.24; Liberty Bonds $ 400; owes a mortgage note for $ 9600; interest on this note for one year at 5 %; owes General Merchandise Emporium $ 117.45; owes the Grange Harvester Co. $ 98.45.

245. Solvency.—If the resources are greater than the liabilities, the debts can all be paid and the business is said to be **solvent.** If its liabilities are greater than the resources, all of the debts cannot be paid and the business is said to be **insolvent** or **bankrupt.** When the liabilities become very much greater than the resources, the business is often placed by the court in the hands of some one called a **receiver.** The receiver closes out the business and pays to each creditor the per cent of his bill that the resources are a per cent of the liabilities.

EXERCISES

1. State which of the following firms are solvent:

	A	B	C	D
Resources,	$ 4560	$ 12,450	$ 13,500	$ 975
Liabilities,	3150	15,600	10,900	725

2. The resources of a firm are $ 5650 and the liabilities $ 7425. How much does it lack of paying all debts ?

3. What per cent of its debts can the firm of Ex. 2 pay to its creditors ?

4. State by an equation in terms of resources, liabilities, and expenses of closing out a business, what part of the debts will be paid. Also, express by an equation how much of any bill payable will be paid.

5. The Wampum Dry Goods Co. was closed out by a receiver at $ 5670.34. If the expenses connected with closing out the business were $ 845.73, what were the net resources? If the total liabilities were $ 8325.87, what per cent of the debts were paid ? How much will the receiver be able to pay of its bill of $ 356.82 to the Tower Manufacturing Company ?

XIX

PARTNERSHIPS

246. Partnerships.—Two or more persons entering into an agreement to carry on some business together form a **partnership** or **firm.** The agreement stipulates the investment of each partner, the activities of each partner, and the distribution among the partners of any profits. Often one partner furnishes all or part of the capital without entering into the activities of the firm. Sometimes such a partner desires that his connection with the firm is not to become known and his name does not appear with that of the firm. Thus, Black, Brown, and White may enter into a partnership to be known as Black, Brown, and Company. White is then called a **silent partner.**

If a partner desires to withdraw from the firm at any time, he must either sell his holding to some one agreeable to the other members, to the other members themselves, or the business as a whole must be sold.

247. Liabilities.—In most states each partner is individually responsible for any and all debts of the firm, regardless of any agreement between the partners. Suppose that a partnership with several partners becomes bankrupt and that only one partner has any private funds. This partner must pay all of the debts, or as far as his resources permit, even though the failure of the firm was brought about through an act of one or more of the other partners.

248. Dividing Profits and Losses.—Suppose Mr. Green, who is earning $ 40 per week, invests $ 3000 in a partnership with Mr. Grey, who is earning $ 30 per week and invests $ 5000. How are they to divide any profits at the end of the year? Together they earn $ 70 per week. As Mr. Green earns $\frac{4}{7}$ of their joint weekly income, should he receive $\frac{4}{7}$ and Mr. Grey $\frac{3}{7}$ of any profits? Would this be just to Mr. Grey, who has furnished $\frac{5}{8}$ of the capital? Or should they base the division of profits on capital invested, thus giving Mr. Green $\frac{3}{8}$ and Mr. Grey $\frac{5}{8}$ of the profits? Would this be just to Mr. Green, who has the higher earning capacity?

There are several plans to which both partners may be willing to agree, of which the following is one: The partnership may give each partner a note for the amount he invested bearing a current rate of interest. This interest becomes then an expense to the partnership. The firm can further pay each partner weekly what he earned or can earn. Any further profits can finally be divided evenly or according to some other agreed ratio.

249. Withdrawals of Funds.—Small withdrawals, either in cash or in the form of purchases from the firm, are merely debited to that partner's account with the firm. If the withdrawal is large, the partner may give the firm a note for the time he will hold the money out of the business, or his percentage of the share of profits may be altered.

EXERCISES

1. Three men decided to go into partnership and buy out a dry-goods business. The stock on hand invoiced at $ 12,350 and the firm selling the business asked $ 3500 for their good-will. The partners decided further to invest $ 7000 for carrying on the business. If the partners make *equal investments*, what will this be for each?

2. Suggest what you would consider a fair division of any gains by Mr. Green and Mr. Grey, mentioned in Art. 248, if they formed a partnership. State this by an equation, using literal numbers.

3. Suppose that they entered into a partnership investing $ 4850 in a stock of goods. How much money had they left with which to carry on their business ?

At the end of a year of business their stock of goods invoices at $ 9835 and they have bills receivable to the amount of $ 2783. The firm owes Mr. Green $ 725 on uncollected salary and a note of $ 3000, together with interest at 6 %; it owes Mr. Grey $ 465 uncollected salary and a note of $ 5000, together with interest at 6 %; bills to an amount of $ 1085.

4. Not counting profits, how much does the firm owe Mr. Green ? Mr. Grey ?

5. Make out a statement showing the resources and the liabilities of the firm. The net present worth of the firm is really the yearly profit. What is this ?

6. Divide the yearly profit according to the manner which you suggested for Ex. 2. What is then the total present worth of each of the partners ?

Mr. Jelks owns 80 A. which he leases to Mr. Granger to plant in wheat on the following conditions: the owner pays for the seed and receives $\frac{2}{5}$ of the grain.

7. If 5 pecks are planted per acre, what will be the cost of the seed at $ 2.65 per bushel ?

8. If 1127 bu. were harvested and sold at $ 1.47 per bushel, what were the returns of each man ? What were the net returns of the owner ?

9. Answer the questions in Exs. 7 and 8 with literal numbers.

10. Suggest and solve some other partnership problems.

CORPORATIONS

250. Organization.—Suppose that a few men desire to enter into some business enterprise, as building a railroad, which requires much more capital than they possess. The customary way is for them to organize a **corporation,** also called a **stock company.** These men are called the **organizers** or **promoters** of the corporation. First, they draw up a statement of the name of the corporation, its officers and their duties, the purpose of its business, the amount of capital to be invested, and the number of equal parts, called **shares of stock,** into which the capital is to be divided. As soon as these men have found a sufficient number of persons willing to invest money in the enterprise, usually fixed by law, they present the above-mentioned statement to the secretary of state in the state in which they are to carry on their business. He issues to them a **charter,** also called **papers of incorporation,** stating the conditions under which the corporation may do business.

The value of a share of stock as stated in the charter, called its **par value,** may be of any amount. In smaller corporations, as in a local creamery, it may be from $ 5 to $ 50; in the larger corporations, as in railroads, it is usually $ 100. At stockholders' meetings each stockholder has a voting power proportional to the number of shares owned. Thus, one owning 51 % of the total stock can control the corporation. How ? The capital equals the par value per share times the number of shares. Thus,

$$C = N \times V_p.$$

251. Corporations versus Partnerships.—Corporations possess many advantages over partnerships, of which the principal ones are:

1. The number of shareholders may be increased to any number so as to secure as large funds as needed.

2. Entering or leaving a partnership is often difficult. Buying or selling shares of stock is a simple matter.

3. In a partnership each partner is individually liable for all debts of the firm. In a corporation each stockholder is usually liable for debts to a sum equal to the par value of the stocks owned.

EXERCISES

1. State three illustrations of corporations, of which at least one is local; that is, in your community.

2. Solve the equation on page 214 for N. A company is incorporated for $10,000, with shares at $50 each. How many shares does it have?

3. How many shares must one person own to hold a controlling interest in the corporation of Ex. 2?

4. In order to create an interest in a Community Fair, it was incorporated for $20,000, with shares of par value $10. How many shares were there? If the charter further provided that no one could own more than $200, par value, of stocks, what is the smallest number of stockholders the fair could have? What reason can you give for this last requirement?

5. Draw up a statement for the incorporation of a company to conduct an electric railway in your city.

6. Draw up a statement for the incorporation of the *athletic* association of your school.

252.   Profitable and Non-Profitable Stocks.—A success-ful corporation has profits, called **dividends,** to distribute among its stockholders.   Dividends are stated as a certain per cent of the **par value** of the stock.   They may be paid yearly, half-yearly, or quarterly.   If a corporation loses money, it **assesses** its stockholders a per cent of the par value of the stock sufficient to pay the deficit.

EXERCISES

1.   What is the yearly income from 5 shares, par value $ 1000 each, which declares—pays—a semiannual dividend of 3 % ? of $1\frac{1}{4}$ % ?

2.   State by an equation the per cent of dividends paid by a corporation in terms of its capitalization and profits. What per cent of dividends does a company incorporated for $ 50,000 pay in distributing $ 1250 of profits ?

3.   A company incorporated for $ 125,000 declares $1\frac{1}{2}$ % quarterly dividends.   What are the yearly profits it dis-tributes among its stockholders ?

4.   If a corporation declared $2\frac{1}{2}$ % dividends quarterly, what interest would that be on money invested in it ? What would be the income from eight $ 50 shares ?

5.   Which brings the better income, a $5\frac{1}{2}$ % mortgage or shares of stock declaring $1\frac{1}{4}$ % dividends quarterly ?

6.   Mr. Jowel bought some shares of stock that paid him 9 % dividends the first year he owned them.   Through changes in conditions the company did not prosper the second year and he had to pay an assessment of 4 % on his shares of stock.   What per cent of interest did he get for the 2 yr. ?   How much per year ?   Compare this with a 5 % mortgage.   Which would you rather own ?

253. Common and Preferred Stocks.—Some corporations issue two forms of shares of stock, **common** (c) and **preferred** (p). While various charters may present a great many differences between common and preferred stocks, the usual difference is that the preferred stocks pay dividends out of the profits up to a certain specified per cent of their par value before any dividends are paid to the holders of common stock.

254. Buying and Selling Stocks.—If a corporation is prosperous, it will soon pay good dividends. Why? Stocks upon which high dividends are paid will be in demand and hence sell **above par.** Why? What stocks would sell **below par?**

Shares of stock are bought and sold upon the **stock exchange** by **stock brokers.** See page 165. The charge for buying as well as for selling shares of stock is $\frac{1}{8}$ % of par value.

The following list gives the quotations of a few stocks as they sold on the New York Stock Exchange one day. The quotations are in per cents of the par value:

		Week before	Year before
Am Beet Sugar.................	$55\frac{1}{2}$	$53\frac{1}{4}$	75
Am Tel & Tel.................	$104\frac{1}{8}$	$104\frac{3}{4}$	$107\frac{1}{4}$
Anaconda Min.................	$65\frac{7}{8}$	$65\frac{1}{4}$	$56\frac{1}{2}$
A T & S F c...................	$94\frac{1}{8}$	93	83
A T & S F p...................	$87\frac{7}{8}$	88	82
Baldwin Loco.................	$90\frac{1}{2}$	$92\frac{3}{4}$	$71\frac{1}{2}$
Erie..........................	$15\frac{1}{4}$	$15\frac{1}{2}$	$22\frac{3}{4}$
Maxwell Motor	32	28	$43\frac{1}{2}$
S P	$83\frac{1}{2}$	$83\frac{3}{4}$	$91\frac{1}{4}$
Wabash c.....................	$9\frac{5}{8}$	$9\frac{1}{2}$	$8\frac{1}{8}$
Wabash p.....................	$38\frac{1}{4}$	$38\frac{1}{8}$	40
West Union...................	$90\frac{3}{4}$	$88\frac{1}{4}$	83
West Electric.................	$42\frac{3}{4}$	$42\frac{1}{2}$	$38\frac{1}{8}$

EXERCISES

1. A corporation was organized with $ 100,000 of common stock and $ 50,000 of preferred stock. The preferred stock was to receive dividends first from any profits up to 5 %, and any remaining profits were to be distributed among the holders of the common stock. What per cent of the par value of their stocks would be received in dividends by the holders of each kind of stock when the corporation distributed a yearly profit of $ 1750 ? of $ 11,500 ?

2. Which are generally the more valuable, common or preferred stocks ? Could there ever be any variation to this rule ?

Answer the Exs. 3–8 from the quotations on the opposite page. Consider $ 100 as par value of each share of stock.

3. Find the cost of 5 shares Anaconda Mining stock.

4. Find returns from selling 12 shares Wabash common.

5. Find the cost of 7 shares Western Union.

6. Find returns from the sale of 4 shares American Telephone and Telegraph.

7. What would the stocks of Ex. 6 have cost at the year-before quotations ? What would have been gained or lost by buying 5 shares then and selling at the present quotations on page 218 ?

8. What would have been gained or lost in buying 12 shares American Beet Sugar stocks at the year-before price and selling them at the present one on page 218 ?

9. Make up and solve some other problems from the data on page 218.

10. Look up the stock quotations in your papers for the present. Compare these with those on page 218. How can you account for the changes found ?

11. Make up some problems and solve from the data on page 218 and from the present stock quotations.

255. Income from Stocks.—Some buy stocks as a speculation in the hope that they will advance in price. Since prices go down as often and as much as they go up, a speculator is as liable to lose as he is to gain. The occasional buyer is even more liable to lose than to gain, as he is unfamiliar with the conditions affecting prices of stocks. There is only one good rule to follow: **Never speculate.**

Stocks are also bought as investments. Even here extreme care must be exercised in their purchase. The advice of some one familiar with any stock of which the purchase is contemplated should be secured. Note that incomes from stocks vary with the profits of the corporation. Explain fully.

256. Comparing Stocks and Bonds.—The bonds issued by a corporation are notes and as such bear a **definite rate of income.** Dividends upon the shares of stock vary from time to time, depending upon the profits of the company.

Bonds are generally secured by a mortgage on the property of the company. If the company becomes bankrupt, the bondholders first get all returns from the sale of the property of the corporation. If the property sells for more than the indebtedness, the stockholders get any surplus, which is usually very little. If the sale of the property is insufficient to pay the indebtedness, the shareholders are usually liable for all debts of the corporation to an amount equal to the par value of the stock that they own.

EXERCISES

1. What will be the cost of a share of stock, par value $1000, quoted at 89⅞ ? What will the one selling the share receive for it ?

2. If the share of stock in Ex. 1 pays a yearly dividend of 5 %, what will be the yearly income from it ?

3. State by an equation, the rate of interest derived from stocks that pay a yearly dividend of d dollars, upon a par value of p dollars, bought at b dollars. At the price found in Ex. 1 and the income found in Ex. 2, what is the rate of interest—that is, what is the rate of income?

4. Stocks bought at $67\frac{3}{8}$ pay yearly dividends of 2.75 %. Find the rate of interest the money invested in them brings.

5. Would it be better to have money invested in secure farm mortgages at $5\frac{1}{2}$ % interest or in stocks giving 8 % dividends bought on the market at $148\frac{1}{8}$?

6. Compare the income of stocks bought at $56\frac{3}{8}$, which declare a yearly dividend of $2\frac{1}{4}$ %, with a 5 % mortgage.

7. Mr. A. bought 12 shares of stock of a certain railroad at $98\frac{3}{4}$ and sold them a year later at $67\frac{1}{4}$. If the par value of the shares was $ 1000, what did he lose?

8. Mr. K. bought 9 shares of oil stock—par value $ 100— at $82\frac{1}{4}$, which paid a yearly dividend of 2 %. He sold them a year later at $46\frac{3}{8}$. What did he lose, not counting interest on the money invested?

9. How much interest did Mr. K.—Ex. 8—lose on the money he had invested in stocks?

10. Mr. Q. bought stocks at $17\frac{3}{8}$ in a corporation that went into the hands of receivers. In settling up the indebtedness of the company Mr. Q. had to pay one assessment of 15 % and another of 7 % of the par value of the stock he owned. If he owned 9 shares—par value $ 100—how much did he lose in all?

11. Would you rather own stocks or bonds in the average corporation? State your reasons fully.

12. Find the cost of 7 Erie 4 % bonds at $53\frac{3}{8}$, par value $ 1000.

13. What is the yearly income from the bonds in Ex. 12? What rate of interest do they pay?

14. Mr. Kayson and Mr. Royser have equal interests in a partnership with resources $ 15,450 and liabilities $ 2875. If they incorporate with a capitalization of $ 15,000, each to own 50 % of the stock, how much must each add ?

15. If each share of stock in Ex. 14 was made $ 50, how many shares would each own ?

16. If they had incorporated their business at $ 15,000 capital with $ 50 shares so that Mr. Kayson is to own 60 % of the stock, how much must Mr. Kayson have invested additionally and how much would have been paid to Mr. Royser ?

17. Find the cost of 12 City of Paris 6 % bonds at $94\frac{3}{4}$, par value $ 1000.

18. What is the yearly income from the bonds in Ex. 17 ? What rate of interest do they pay on the investment ?

19. The S. S. Bank of Chicago pays 20 % interest on the capital stock. When its shares of stock sell for 404, what rate of interest do investments in these stocks pay ?

20. Find the cost of $ 1200 Liberty Bonds $3\frac{1}{2}$ % if bought at 101.84 with .125 brokerage.

21. What will be the yearly income from the bonds in Ex. 20 ? What rate of interest will they pay on the investment ?

22. Why is the income rate from bonds in Ex. 20 lower than that from those in Ex. 19, which again bring a lower income than the bonds in Ex. 17 ?

23. Why will the price of stocks in a corporation vary much more than bonds of the same corporation ?

24. Compare the rate of income from a 5 % mortgage that is well secured with that of stocks bought at $96\frac{7}{8}$ which declare the following yearly dividends for 6 years: 7 %, *5.5 %, 5 %,* 3.5 %, 3 %, 2 %.

REFERENCE TABLES

ENGLISH SYSTEM

Length

12 inches (in.) = 1 foot (ft.).

3 feet = 1 yard (yd.).

5¼ yards, or 16½ feet = 1 rod (rd.).

320 rods, or 5280 feet = 1 mile (mi.).

5 ft. 3 in. may be written 5′ 3″.

SURVEYORS' TABLE OF LENGTH

7.92 inches = 1 link (li.).

100 links = 4 rods = 1 chain (ch.).

80 chains = 5280 ft. = 1 mile.

ADDITIONAL UNITS OF LENGTH

4 inches = 1 hand.

6 feet = 1 fathom.

100 fathoms = 1 cable length.

1.15 common miles = 1 knot (nautical mile).

Square Measure

144 square inches (sq. in.) = 1 square foot (sq. ft.).

9 square feet = 1 square yard (sq. yd.).

30¼ square yards = 1 square rod (sq. rd.).

160 square rods = 1 acre (A.).

640 acres = 1 square mile (sq. mi.).

SURVEYORS' TABLE OF SQUARE MEASURE

16 square rods (sq. rd.) = 1 square chain (sq. ch.).

10 square chains = 1 acre (A.).

640 acres = 1 square mile (sq. mi.).

36 square miles = 1 township (tp.).

223

Cubic Measure

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.).
27 cubic feet = 1 cubic yard (cu. yd.).
128 cubic feet = 1 cord (cd.).
24¾ cubic feet = 1 perch (stone, etc.).
A cubic yard is called a load.

Weight

AVOIRDUPOIS

16 ounces (oz.) = 1 pound (lb.).
100 pounds = 1 hundredweight (cwt.).
2000 pounds = 1 ton (T.).
112 pounds = 1 long hundredweight.
2240 pounds = 1 long ton.

Long ton is used in U. S. Custom-house and at mines.
1 ton soft coal occupies about 35 cubic feet.
1 ton hard coal occupies about 28 cubic feet.

Liquid Measure

4 gills (gi.) = 1 pint (pt.).
2 pints = 1 quart (qt.).
4 quarts = 1 gallon (gal.).
231 cubic inches = 1 gallon.
31.5 gallons = 1 barrel (bbl.) (varies).
2 barrels = 1 hogshead (varies).
1 pint = 16 fluid ounces (apothecaries').
57.75 cubic inches = 1 liquid quart.
½ pint = 1 measuring cup.

1 cubic foot of water weighs nearly 62½ pounds.
1 gallon of water weighs nearly 8⅓ pounds.
1 cubic foot of water equals about 7½ gallons.

Dry Measure

2 pints (pt.) = 1 quart (qt.).
8 quarts = 1 peck (pk.).
4 pecks = 1 bushel (bu.).
2150.4 cubic inches = 1 bushel.
1 stricken bushel = 1¼ cubic feet (nearly).
1 heaped bushel = 1½ cubic feet (nearly).
1 bushel ear corn = 2½ cubic feet (nearly).

THE FOLLOWING HOLD IN NEARLY ALL STATES

1 bushel of wheat weighs 60 pounds.
1 bushel of shelled corn weighs 56 pounds.
1 bushel of ear corn weighs 75 or 80 pounds in the fall and 70 pounds later.
1 bushel of oats weighs 32 pounds.
1 bushel of rye weighs 56 pounds.
1 bushel of barley weighs 48 pounds.
1 bushel of potatoes weighs 60 pounds.
1 bushel of beans weighs 60 pounds.
1 bushel of peas weighs 60 pounds.
1 bushel of apples weighs 48 pounds.
1 bushel of clover seed weighs 60 pounds.
1 bushel of alfalfa seed weighs 60 pounds.
1 bushel of timothy seed weighs 45 pounds.
1 bushel of bran weighs 20 pounds.
1 bushel of soft coal weighs 80 pounds.
1 barrel of flour weighs 196 pounds.
1 barrel of pork or beef weighs 200 pounds.

Angles and Arcs

60 seconds (60″) = 1 minute (1′).
60 minutes = 1 degree (1°).
90 degrees = 1 right angle.
360 degrees = 1 circumference.

Time

60 seconds (sec.) = 1 minute (min.).
60 minutes = 1 hour (hr.).
24 hours = 1 day (da.).
7 days = 1 week (wk.).
12 months (mo.) = 1 year (yr.).
365 days = 1 common year.
366 days = 1 leap year.

United States Money

10 mills = 1 cent (ct. or ¢).
10 cents = 1 dime (d.).
10 dimes = 1 dollar ($).
10 dollars = 1 eagle (E.).

Counting

12 units = 1 dozen (doz.).
12 dozen, or 144 = 1 gross (gr.).
12 gross, or 1728 = 1 great gross.
20 units = 1 score.

The dozen is being replaced by 10 and the gross by 100.

500 sheets of paper are called a ream.

METRIC SYSTEM

Length

10 millimeters (mm.) = 1 centimeter (cm.).
10 centimeters = 1 decimeter (dm.).
10 decimeters = 1 meter (m.).
10 meters = 1 Dekameter (Dm.).
10 Dekameters = 1 Hektometer (Hm.).
10 Hektometers = 1 Kilometer (Km.).

Square Measure

100 square millimeters (mm².) = 1 square centimeter (cm².).
100 square centimeters (cm².) = 1 square decimeter (dm².).
100 square decimeters = 1 square meter (m².).
100 square meters = 1 square Dekameter (Dm².).
100 square Dekameters = 1 square Hektometer (Hm².).
100 square Hektometers = 1 square Kilometer (Km².).

A square Dekameter is called an **are**. As 100 square Dekameters equals 1 square Hektometer, a square Hektometer is called a **hektare** (ha.). These are the metric units of land measure.

Cubic Measure

1000 cubic millimeters (mm³.) = 1 cubic centimeter (cm³.).
1000 cubic centimeters = 1 cubic decimeter (dm³.).
1000 cubic decimeters = 1 cubic meter (m³.).
1000 cubic meters = 1 cubic Dekameter (Dm³.).
1000 cubic Dekameters = 1 cubic Hektometer (Hm³.).
1000 cubic Hektometers = 1 cubic Kilometer (Km³.).

The cubic meter is used in measuring wood and is called the **stere** (st.).

Weight

$$10 \text{ milligrams (mg.)} = 1 \text{ centigram (cg.).}$$
$$10 \text{ centigrams} = 1 \text{ decigram (dg.).}$$
$$10 \text{ decigrams} = 1 \text{ gram (g.).}$$
$$10 \text{ grams} = 1 \text{ Dekagram (Dg.).}$$
$$10 \text{ Dekagrams} = 1 \text{ Hektogram (Hg.).}$$
$$10 \text{ Hektograms} = 1 \text{ Kilogram (Kg.).}$$
$$1000 \text{ Kilograms} = 1 \text{ Metric ton (t.).}$$

The gram is the weight of 1 cm³. of water at a temperature of 4° C.

Capacity

$$10 \text{ milliliters (ml.)} = 1 \text{ centiliter (cl.).}$$
$$10 \text{ centiliters} = 1 \text{ deciliter (dl.).}$$
$$10 \text{ deciliters} = 1 \text{ liter (l.).}$$
$$10 \text{ liters} = 1 \text{ Dekaliter (Dl.).}$$
$$10 \text{ Dekaliters} = 1 \text{ Hektoliter (Hl.).}$$
$$10 \text{ Hektoliters} = 1 \text{ Kiloliter (Kl.).}$$

The liter is 1 dm³.

Equivalents

A meter	= 39.37 in.	= 3¼ ft. (nearly).
A Kilometer	= .621 mi.	= .6 mi. (nearly).
A liter	= 1.056 qt. (liquid)	= 1 qt. (nearly).
A liter	= .908 qt. (dry)	= .9 qt. (nearly).
A Kilogram	= 2.204 lb.	= 2.2 lb. (nearly).
A hectare	= 2.47 A.	= 2½ A. (nearly).

Formulas of Areas and Volumes

Area parallelogram	$= ab.$
Area triangle	$= \frac{1}{2} ab.$
Area trapezoid	$= \frac{1}{2} a(B + b).$
Area circle	$= \pi R^2.$
Area ring	$= \pi (R^2 - r^2).$
Circumference circle	$= 2 \pi R.$
Lateral surface regular pyramid	$= \frac{1}{2} nls.$
Lateral surface cone	$= \pi Rs.$
Surface sphere	$= 4 \pi R^2.$
Volume pyramid	$= \frac{1}{3} Bh.$
Volume cone	$= \frac{1}{3} \pi R^2 h.$
Volume sphere	$= \frac{4}{3} \pi R^3.$

TABLE OF SQUARE ROOTS

2	1.4142	26	5.0990	51	7.1414
3	1.7321	29	5.3852	53	7.2801
5	2.2361	30	5.4772	55	7.4162
6	2.4495	31	5.5678	57	7.5498
7	2.6458	33	5.7446	58	7.6158
10	3.1623	34	5.8310	59	7.6811
11	3.3166	35	5.9161	61	7.8102
13	3.6056	37	6.0828	62	7.8740
14	3.7417	38	6.1644	65	8.0623
15	3.8730	39	6.2450	66	8.1240
17	4.1231	41	6.4031	67	8.1854
19	4.3589	42	6.4807	68	8.2462
21	4.5826	43	6.5574	69	8.3066
22	4.6904	46	6.7823	70	8.3666
23	4.7958	47	6.8557	71	8.4261

$$\pi = 3.1416 \quad 1.7725$$

TABLE OF CUBE ROOTS

2	1.2599	21	2.7589	42	3.4760
3	1.4422	22	2.8020	43	3.5034
4	1.5874	23	2.8439	44	3.5303
5	1.7100	25	2.9240	45	3.5569
6	1.8171	26	2.9625	46	3.5830
7	1.9129	28	3.0366	47	3.6088
9	2.0801	29	3.0723	49	3.6593
10	2.1544	30	3.1072	50	3.6840
11	2.2240	31	3.1414	51	3.7084
12	2.2894	33	3.2075	52	3.7325
13	2.3513	34	3.2396	53	3.7563
14	2.4101	35	3.2711	55	3.8030
15	2.4662	36	3.3019	57	3.8485
17	2.5713	37	3.3322	58	3.8709
18	2.6207	38	3.3620	59	3.8930
19	2.6684	39	3.3912	60	3.9149
20	2.7144	41	3.4482	61	3.9365

TABLES OF LOGARITHMS

The following two pages contain tables of the decimal part of logarithms of numbers having three digits or less.

To find the logarithm of the decimal part of a number, as 356, look down the left-hand column until 35 is reached, then to the right to the column having 6 at the top, where 5514 is found as the logarithm of 356. The logarithm of 72 is the same as of 720. The logarithm of 8 is the same as of 800. To find the logarithm of 3564, find .4 of the difference between the logarithms of 357 and 356, then add this to the logarithm of 356. Hence, the logarithm of 3564 is 5514 + 5 = 5519.

To find the number having a logarithm that does not appear in the tables, first find the number whose logarithm is next smaller. Find the difference between this next smaller and the given logarithm. Also, find the difference between the next smaller and the next larger logarithm. Multiply the smaller difference by 100, divide by the larger difference, and annex the digits to the number with the next smaller logarithm. In finding the number whose logarithm is 3309, note that 3304 is next smaller and their difference is 5. The difference between 3304 and the next larger logarithm is 20. Also, 500 ÷ 20 = .25, to be annexed to 214, the number having the logarithm 3304. This gives the required number 21425.

Use the straight edge of a card or paper to follow the line across the page, to avoid errors.

N	0	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	·7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396

N	0	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	2677
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996

RATIOS OF ANGLES

Explanation of Table of Ratios.—If a right triangle is drawn with one angle 40°, the other acute angle will be 50°. It is seen from this triangle that the sine of 40° equals the cosine of 50°. Similarly, the tangent of 40° equals the cotangent of 50°. Again, the sine of 50° equals the cosine of 40°. That is, any ratio equals the co-ratio of the **complementary** angle. The sine of 30° equals the cosine of 60°; the cotangent of 70° equals the tangent of 20°.

Tables of ratios of angles are usually made for the first 45° only. Ratios of an angle between 45° and 90° are found from the co-named ratios of the **complementary angle.** Thus, to find sin 63° we find cos 27°. Similarly, for tan 49° we find cot 41°; for cos 81° we find sin 9°. So, to find a ratio of an angle between 45° and 90°, first find its complement and then find the co-named ratio in the tables.

In using the tables on the opposite page, note that when the angle is found in the column to the **left** the names of the ratios are given at the **top**. To find tan 36°, begin at the top of the column marked **tan** and go **down** until opposite 36 in the column to the left, where is found 0.7265 as the value of tan 36°. When the angle is found in the column to the **right** the names of the ratios are given **below** the columns. To find sin 76°, begin at the bottom of the column marked **sin** and go **up** until opposite 76° in the column to the right, where is found 0.9703 as the value of sin 76°.

The tables on the opposite page give only the ratios of angles of whole degrees, which is sufficient for our purposes. In higher mathematics and in engineering much larger tables are used, which give ratios of angles for minutes and seconds of arc.

	sin	cos	tan	cot	
0°	0.0000	1.0000	0.0000	————	90°
1°	0.0175	0.9998	0.0175	57.2900	89°
2°	0.0349	0.9994	0.0349	28.6363	88°
3°	0.0523	0.9986	0.0524	19.0811	87°
4°	0.0698	0.9976	0.0699	14.3007	86°
5°	0.0872	0.9962	0.0875	11.4301	85°
6°	0.1045	0.9945	0.1051	9.5144	84°
7°	0.1219	0.9925	0.1228	8.1443	83°
8°	0.1392	0.9903	0.1405	7.1154	82°
9°	0.1564	0.9877	0.1584	6.3138	81°
10°	0.1736	0.9848	0.1763	5.6713	80°
11°	0.1908	0.9816	0.1944	5.1446	79°
12°	0.2079	0.9781	0.2126	4.7046	78°
13°	0.2250	0.9744	0.2309	4.3315	77°
14°	0.2419	0.9703	0.2493	4.0108	76°
15°	0.2588	0.9659	0.2679	3.7321	75°
16°	0.2756	0.9613	0.2867	3.4874	74°
17°	0.2924	0.9563	0.3057	3.2709	73°
18°	0.3090	0.9511	0.3249	3.0777	72°
19°	0.3256	0.9455	0.3443	2.9042	71°
20°	0.3420	0.9397	0.3640	2.7475	70°
21°	0.3584	0.9336	0.3839	2.6051	69°
22°	0.3746	0.9272	0.4040	2.4751	68°
23°	0.3907	0.9205	0.4245	2.3559	67°
24°	0.4067	0.9135	0.4452	2.2460	66°
25°	0.4226	0.9063	0.4663	2.1445	65°
26°	0.4384	0.8988	0.4877	2.0503	64°
27°	0.4540	0.8910	0.5095	1.9626	63°
28°	0.4695	0.8829	0.5317	1.8807	62°
29°	0.4848	0.8746	0.5543	1.8040	61°
30°	0.5000	0.8660	0.5774	1.7321	60°
31°	0.5150	0.8572	0.6009	1.6643	59°
32°	0.5299	0.8480	0.6249	1.6003	58°
33°	0.5446	0.8387	0.6494	1.5399	57°
34°	0.5592	0.8290	0.6745	1.4826	56°
35°	0.5736	0.8192	0.7002	1.4281	55°
36°	0.5878	0.8090	0.7265	1.3764	54°
37°	0.6018	0.7986	0.7536	1.3270	53°
38°	0.6157	0.7880	0.7813	1.2799	52°
39°	0.6293	0.7771	0.8098	1.2349	51°
40°	0.6428	0.7660	0.8391	1.1918	50°
41°	0.6561	0.7547	0.8693	1.1504	49°
42°	0.6691	0.7431	0.9004	1.1106	48°
43°	0.6820	0.7314	0.9325	1.0724	47°
44°	0.6947	0.7193	0.9657	1.0355	46°
45°	0.7071	0.7071	1.0000	1.0000	45°
	cos	sin	cot	tan	

ABBREVIATIONS USEFUL IN BUSINESS

acct. or a/c	account of
agt. . . .	agent
a/s	account sales
@	at; each; to
bal. . . .	balance
bdl.	bundle
bl.	bale
B/L . . .	bill of lading
bot. . . .	bought
C	one hundred
chg. . . .	charge
ck. or √ .	check; correct
cml. . . .	commercial
c/o	care of
c.o.d. . . .	collect on delivery
coll. . . .	collection
com. . . .	commission
consg't. . .	consignment
cr.	credit; crate
cs.	case
dept. . . .	department
dft. . . .	draft
Dr.	debit; doctor
ea.	each
e.o.a. . . .	error and omission accepted
exch. . . .	exchange
f.o.b. . . .	free on board
ft.	freight
gro.	gross
guar. . . .	guaranty; guarantee
ins.	insurance

inst.. . . .	instant; present month
int.	interest
I; inv. . . .	invoice
inv't. . . .	inventory
l.p.	list price
ltd.	limited
M	one thousand
mdse. . . .	merchandise
mortg. . .	mortgage
O.K. . . .	all correct
o/d	on demand
pay't . . .	payment
pc.	piece
pd.	paid
per	by; by the
pfd. . . .	preferred
pkg. . . .	package
pr.	pair
prox. . . .	following month
rec'd. . . .	received
rec't. . . .	receipt
sec'y. . . .	secretary
set.	settlement
ship. . . .	shipment
sig.	signed; signature
stk.	stock
treas. . . .	treasurer; treasury
ult.	last month
via	by way of
viz.	namely; to wit
wt.	weight
#	No., before figures; lb., after figures

TABLE OF LEGAL AND MAXIMUM RATES
OF INTEREST

	Legal Rate	Maximum Permitted		Legal Rate	Maximum Permitted
Alabama	8	8	Montana	8	12
Alaska	8	12	Nebraska	7	10
Arizona	6	10	Nevada	7	12
Arkansas	6	10	New Hampshire	6	6
California	7	any	New Jersey	6	6
Colorado	8	any	New Mexico	6	12
Connecticut	6	6	New York	6	6
Delaware	6	6	North Carolina	6	6
District Columbia	6	10	North Dakota	6	10
Florida	8	10	Ohio	6	8
Georgia	7	8	Oklahoma	6	10
Idaho	7	12	Oregon	6	10
Illinois	5	7	Pennsylvania	6	6
Indiana	6	8	Rhode Island	6	any
Iowa	6	8	South Carolina	7	8
Kansas	6	10	South Dakota	7	12
Kentucky	6	6	Tennessee	6	6
Louisiana	5	8	Texas	6	10
Maine	6	any	Utah	8	12
Maryland	6	6	Vermont	6	any
Massachusetts	6	any	Virginia	6	6
Michigan	5	7	Washington	6	12
Minnesota	7	10	West Virginia	6	6
Mississippi	6	8	Wisconsin	6	10
Missouri	6	8	Wyoming	8	12

The penalty for usury varies greatly. It ranges from none where there is no maximum fixed rate up to forfeiture of both principal and interest.

Legal rates are collected where interest has been agreed upon without specifying the rate. When any indebtedness begins to draw interest through some process of law, as in the case of a judgment, the rate is always the legal rate. The latter is the most usual occurrence of legal rates.

INDEX

237